PCEL-4302
ELECTRICAL MACHINES-II

5TH SEMESTER B.TECH IN ELECTRICAL ENGINEERING
MODULE-I  
(15 HOURS)
1. Three Phase Synchronous Generators (5 hours)
   Synchronous Generator Construction (both Cylindrical Rotor and Salient Pole type), The Speed of Rotation of a Synchronous Generator, Induced voltage in A.C. Machines. The Internal Generated Voltage of a Synchronous Generator, The Effect of Coil Pitch on A.C. Machines, Distributed Windings in A.C. Machines, The Rotating Magnetic Field, The Equivalent Circuit of a Synchronous Generator (Armature Reaction Reactance, Synchronous Reactance and Impedance). [Chapman: Ch. 5.1, 5.2, 4.4, 5.3, B.1, B.2, 4.2, 5.4]  
2. Cylindrical Rotor type Three Phase Synchronous Generators (4+2=6 hours)
   (a) The Phasor Diagram of a Synchronous Generator, Power and Torque in Synchronous Generators (Power Angle Equation and Power Angle Characteristic), Measuring Synchronous Generator Model Parameters (Open Circuit and Short Circuit Tests and Determination of Synchronous Impedance and Reactance, The Short Circuit Ratio), Voltage Regulation and Speed Regulation. [Chapman: Ch. 5.5, 5.6, 5.7, 4.8] (4 hours)
   (b) Zero Power Factor characteristic, Potier Reactance, Voltage Regulation by Synchronous Impedance Method, Potier Reactance (Zero Power Factor = ZPF) Method. [M.G.Say: Selected Portions of Ch.10.2, 10.3, 10.4, 10.15] (2 hours)  
3. Salient Pole type Three Phase Synchronous Generators (3+1=4 hours)
   Two Reaction Concept. Development of the Equivalent Circuit of a Salient Pole type Three Phase Synchronous Generator (Direct axis and Quadrature axis Reactances, Phasor Diagram for various load power factors.), Torque and Power Equations of Salient Pole Synchronous Generator (Power Angle Equation and Power Angle Characteristic with stator resistance neglected). [Chapman: Appendix C.1, C.2] (3 hours)
   Slip Test for determination of Direct axis and Quadrature axis Reactances. [M.G.Say: Ch.10.15] (1 hour)

MODULE-II  
(12 HOURS)
4. Parallel operation of Three Phase A.C. Synchronous Generators (4 hours)
   The Conditions Required for Paralleling, The General Procedure for Paralleling Generators, Frequency - Real Power and Voltage – Reactive Power Characteristics of a Three Phase Synchronous Generator, Operation of Generators in Parallel with large Power Systems, Operation of generators in parallel with other Generators of the same size. [Chapman: Ch.5.9]  
5. Three Phase Synchronous Motors  
   (8 hours)
   Basic Principles of Motor operation, Steady State Synchronous Motor operation, Starting Synchronous Motors, Synchronous Generators and Synchronous Motors, Synchronous Motor Ratings. [Chapman: Ch.6.1, 6.2, 6.3, 6.4, 6.5]  

MODULE-III  
(13 HOURS)
6. Three Phase Transformers (5+3=8 hours)
   Constructional features, Three-Phase Transformer connections, The per unit system for Three Phase Transformer, Transformer Ratings and Related problems, Two Single-Phase Transformers connected in Open Delta (V-Connection) and their rating, T-
Connection (Scott Connection) of Two Single-Phase Transformers to convert Three-Phase balanced supply to Two-Phase balanced supply. [Chapman: Ch.2.10, 2.11, 2.12] (5 hours)

Transformer Three phase Connections: Various Phase Displacements (0°, 180°, +30° and -30°), Connection Diagrams and Phasor Diagrams of various Vector Groups (Yy0, Dd0, Dz0, Yy6, Dd6, Dz6, Yd1, Dy1, Yz1, Yd11, Dy11, Yz11), Parallel operation of three phase transformers. [M.G.Say: Ch.5.9, 5.15] (3 hours)

7. Single Phase and Special Purpose Motors (5 hours)

The Universal Motor, Introduction to Single Phase Induction Motors, Starting of Single Phase Induction Motors, Speed Control of Single Phase Induction Motors, The Circuit Model of a Single Phase Induction Motor, Other types of Motors: Reluctance Motors, Stepper Motors. [Chapman: Ch.10.1, 10.2, 10.3, 10.4, 10.5, 10.6]

TEXT BOOKS:


REFERENCE BOOKS:


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CHAPTER-3

Salient Pole type Three Phase Synchronous Generators
Introduction:

The simple cylindrical theory of a synchronous generator ignores the effect of the reluctance torque on the generator. Fig.3.1 shows a salient-pole rotor with no windings inside a three-phase stator. The stator magnetic field produced in the air gap of the generator induces a magnetic field in the rotor. The flux induced in the rotor will act along the axis of the rotor. Since there is an angle between the stator magnetic field and the rotor magnetic field, a torque will be induced in the rotor which will tend to line up the rotor with the stator field. The magnitude of this torque is proportional to ‘\(\sin(2\delta)\)’ where \(\delta\) is the angle between the two magnetic fields. This torque is known as the ‘reluctance torque’.

Fig.3.1: A salient-pole rotor, illustrating the idea of reluctance torque.

3.1: Development of the Equivalent Circuit of a Salient-Pole Synchronous Generator

There are four elements in the equivalent circuit of a synchronous generator:
1. The resistance of the stator winding
2. The self-inductance of the stator winding
3. The internal generated voltage of the generator \(E\)
4. The armature reaction of the synchronous generator

The first three elements as above remain same as in the case of cylindrical rotor theory of synchronous generators, but the 4th term representing the armature-reaction effect must be modified to explain the salient-pole rotor theory. Fig.3.2 shows a two-pole salient-pole rotor rotating anti-clockwise within a two-pole stator. The rotor flux density \(B_R\) points upward. The equation for the induced voltage on a moving conductor in the presence of a magnetic field is

\[
e_{ind} = (v \times B). l
\]

The voltage in the conductors in the upper part of the stator will be positive out of the page, and the voltage in the conductors in the lower part of the stator will be into the page. The plane of maximum induced voltage will lie directly under the rotor pole at any given time.
Fig. 3.2: The effects of armature reaction in a salient-pole synchronous generator.

The rotor magnetic field induces a voltage in the stator which becomes maximum in the conductors directly under the pole faces (Fig. 3.2.a). When a lagging load is connected to the generator, a stator current will flow that becomes maximum at an angle behind $E_A$ (Fig. 3.2.b). This stator current $I_A$ produces a stator magneto-motive force (mmf) in the machine (Fig. 3.2.c). The stator mmf produces a stator flux density $B_S$ (Fig. 3.2.d). The reluctance of the direct-axis flux path is lower than the reluctance of the quadrature-axis flux path; as a result, the direct-axis component of mmf produces more flux per ampere-turn than the quadrature-axis component. The direct- and quadrature-axis stator fluxes produce armature reaction voltages in the stator of the machine (Fig. 3.2.e).

If a lagging load is now connected to the terminals of this generator, then a current will flow whose peak is delayed behind the peak voltage. This current is shown in Fig. 3.2b. The stator current flow produces a magneto motive force that lags $90^0$ behind the plane of peak stator current, as shown in Fig. 3.2c. In the cylindrical theory, this mmf then produces a stator magnetic flux density $B_S$ that lines up with the stator mmf. However, it is actually easier to produce a magnetic flux density in the direction of the rotor than it is to produce one in the direction perpendicular to the rotor. Therefore, the stator mmf can be resolved into components parallel to and perpendicular to the axis of the rotor. Each one of these mmf components produces a magnetic field with more flux per ampere-turn being produced along the direct axis as compared to that along the quadrature axis.
Fig. 3.3: The phase voltage of the generator is just the sum of its internal generated voltage and its armature reaction voltages.

The resulting stator magnetic field is shown in Fig. 3.2d, compared to the field predicted by the cylindrical rotor theory. Now, each component of the stator magnetic field produces a voltage of its own in the stator winding by armature reaction. These armature-reaction voltages are shown in Fig. 3.2e. The total voltage in the stator is thus

\[ V_0 = E_A + E_d + E_q \quad 3.2 \]

Here \( E_d \) is the direct-axis component of the armature-reaction voltage and \( E_q \) is the quadrature-axis component of armature reaction voltage (Fig.3.3). Each armature-reaction voltage is directly proportional to its stator current and delayed \( 90^\circ \) behind the stator current. Therefore, each armature-reaction voltage can be modeled as shown in equations 3.3 and 3.4 below.

\[ E_d = j I_d x_d \quad 3.3 \]
\[ E_q = j I_q x_q \quad 3.4 \]

The stator terminal voltage becomes

\[ V_0 = E_A - j I_d x_d - j I_q x_q \quad 3.5 \]

The armature resistance and self-reactance must now be included. The armature self-reactance \( X_A \) is independent of the rotor angle and is normally added to the direct and quadrature armature-reaction reactances to produce the direct axis synchronous reactance and the quadrature axis synchronous reactance of the generator (equations 3.6 and 3.7).

\[ X_d = x_d + X_A \quad 3.6 \]
\[ X_q = x_q + X_A \quad 3.7 \]

Fig. 3.4: The phasor diagram of a salient-pole synchronous generator.

The final expression for the phase voltage of a salient-pole synchronous generator is as mentioned in Eq.3.8 below.

\[ E_A = V_0 + R_A I_A + j I_d X_d + j I_q X_q \quad 3.8 \]

The resulting phasor diagram is shown in Fig. 3.4. In this phasor diagram, the armature current needs to be resolved into two components - one in parallel with \( E_A \) and the other in quadrature.
with $E_A$. However, the angle $(\delta + \theta)$ between $E_A$ and $I_A$ is not known before the diagram is constructed. Normally, the power-factor angle $\theta$ is known in advance. But it is possible to construct the phasor diagram without having the advance knowledge of the angle $\delta$, as shown in Fig. 3.5.

\[ E'_A = V_\phi + R_A I_A + j I_A X_q \quad 3.9 \]

The power factor angle $\theta$ between $V_\phi$ and $I_A$ can be found by using information known at the terminals of the generator. Thus the angle $\delta$ can be determined with knowledge of the armature current $I_A$, power factor angle $\theta$, armature resistance $R_A$ and armature quadrature axis reactance $X_q$. Once the angle $(\delta+\theta)$ is known, the armature current can be broken down into direct and quadrature components and the internal generated voltage can be determined.

### 3.2 TORQUE AND POWER EQUATIONS OF SALIENT-POLE MACHINE

The power output of a cylindrical rotor synchronous generator with negligible stator (armature) resistance as a function of the load angle is given by equation (3.10).

\[ \frac{3E_A V_\phi}{X_s} \sin \delta \quad 3.10 \]

Making the same assumption of negligible stator (armature) resistance the output power of a salient-pole synchronous generator has to be found out as a function of load (torque) angle (Fig. 3.6). The power output of a synchronous generator $P$ is the sum of the power due to the direct-axis current $P_d$ and the power due to the quadrature-axis current $P_q$. Both $I_d$ and $I_q$ contribute to the output power as shown in Fig. 3.6.
Fig. 3.6: Determining the power output of a salient-pole synchronous generator.

\[ P = P_d + P_q = 3V_\phi I_d \cos(90^\circ - \delta) + 3V_\phi I_q \cos \delta \]

\[ = 3V_\phi I_d \sin \delta + 3V_\phi I_q \cos \delta \quad 3.11 \]

From Fig. 3.6, the direct-axis current is given by

\[ I_d = \frac{E_A - V_\phi \cos \delta}{X_d} \quad 3.12 \]

and the quadrature-axis current is given by

\[ I_q = \frac{V_\phi \sin \delta}{X_d} \quad 3.13 \]

Substituting Equations (3.12) and (3.13) into Equation (3.11) yields

\[ P = 3V_\phi \frac{E_A - V_\phi \cos \delta}{X_d} \sin \delta + 3V_\phi \frac{V_\phi \sin \delta}{X_d} \cos \delta \]

\[ = \frac{3V_\phi E_A}{X_d} \sin \delta + 3V_\phi^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin \delta \cos \delta \quad 3.14 \]

\[ = \frac{3V_\phi E_A}{X_d} \sin \delta + \frac{3V_\phi^2}{2} \left( \frac{X_d - X_q}{X_q X_d} \right) \sin 2\delta \quad 3.15 \]

The first term of this expression is the same as the power in a cylindrical rotor machine, and the second term is the additional power due to the reluctance torque in the machine. Since the induced torque in the generator is given by \( T_{ind} = \frac{P_{conv}}{\omega_m} \), the induced torque in the motor can be expressed as

\[ T_{ind} = \frac{3V_\phi E_A}{\omega_m X_d} \sin \delta + \frac{3V_\phi^2}{2\omega_m} \left( \frac{X_d - X_q}{X_q X_d} \right) \sin 2\delta \quad 3.16 \]
The induced torque out of a salient-pole generator as a function of the torque angle $\delta$ is plotted in Fig. 3.7.

Fig. 3.7: The developed torque versus torque angle for a salient-pole synchronous generator.

Example: A 450 volt, 50 Hz, $\Delta$ connected, 4 pole, 3-phase synchronous generator has direct axis reactance of 0.15\(\Omega\) and quadrature axis reactance of 0.07\(\Omega\). Its armature resistance may be neglected. At full load the generator supplies 150A at a power factor of 0.8 lagging.

(a) Find the internal generated voltage $E_A$ of this generator at full load, assuming that it has cylindrical rotor reactance is $X_d$.

(b) Find the generated internal voltage $E_A$ of this generator at full load assuming that it has a salient pole rotor.

Solution

Since the generator is delta connected the armature current $I_A = \frac{150}{\sqrt{3}} = 86.60$ A

Since the power factor is 0.8 the power factor angle is $\cos^{-1} 0.8 = 36.87^\circ$

Therefore internal generated voltage

$E_A = V_\phi + jX_d I_A = 415 <0^0 + j(0.15)(86.60 <-36.87^0) = 428.30 < 9.27^0$

For salient pole we have to consider the quadrature axis reactance.

$E_A' = V_\phi + I_A R_A + I_A jX_q$

$I_d = I_A \sin(\theta+\delta) = 86.6 \sin(36.86+0.66) = 52.74$

Similarly $I_q = I_A \cos(36.86+4.83) = 86.60 \cos(36.60+0.66) = 68.92$

Finally $I_q = 52.74 < - 89.34^0$

$I_q = 68.92 < 0.66^0$

The resulting generated voltage is

$E_A = V_\phi + I_A R_A + jI_d X_d + jI_q X_q$

$= 415 < 0^0 + 0 + (j52.74 < -89.34^0)(0.15) + (j68.92 < 0.66^0)(0.07)$

$= 415 < 0^0 + j7.911 < -89.34^0 + j4.82 < 0.66 = 415 + 7.91 + j0.115 + j4.81 - 0.05552$

$= 422.85 + j4.925 = 422.87 < 0.66^0$

It can be seen that here the magnitude of $E_A$ is not much affected in salient pole but the angle of $E_A$ is much affected in salient pole than the non salient pole machine.
3.3 SLIP TEST

Slip Test is performed in a salient pole synchronous machine to measure the (1) direct axis synchronous reactance and (2) the quadrature axis synchronous reactance of the synchronous machine.

THEORY: - Direct axis synchronous reactance and quadrature axis synchronous reactance are the steady state reactances of the synchronous machine. These reactances can be measured by performing open circuit, short circuit tests and the slip test on synchronous machine.

(A) Direct-axis synchronous reactances \( X_d \):- The direct axis synchronous reactance of synchronous machine in per unit is equal to the ratio of field current \( I_{fSC} \) for rated armature current from the short circuit test, to the field current, \( I_{fOC} \) for rated voltage on the air gap line. Thus, direct axis synchronous reactance \( (X_d) = \frac{I_{fSC}}{I_{fOC}} \) per unit. This direct-axis reactance can be found out by performing open circuit and short circuit test on alternator.

(B) Quadrature-axis synchronous reactance \( X_q \) by slip test:-

For the slip test, the alternator should be driven at a speed, slightly less than the synchronous speed, with its field circuit open. 3 phase balanced reduced voltage of rated frequency is applied to armature (stator) terminals of the synchronous machine. Applied voltage is to be adjusted, so that the current drawn by the stator winding is full load rated current. The stator current and stator voltage undergo changes between minimum and maximum values. When the crest of the stator mmf wave coincides with the direct axis of the rotating field, the induced emf in the open field is zero, the voltage across the stator terminals will be the maximum and the current drawn by the stator winding is the minimum. Thus approximate value of direct axis synchronous reactance \( X_{ds} \) is given by

\[
X_{ds} = \frac{E_{max}}{I_{min}} \quad 3.17
\]

When the crest of stator mmf wave coincides with the quadrature axis of the rotating field, the induced emf in the open circuit field is maximum, the voltage across the stator terminals will be minimum and the current drawn by the stator winding is maximum. Hence, approximate value of the quadrature-axis synchronous reactance; \( X_{qs} \) is given by,

\[
X_{qs} = \frac{E_{min}}{I_{max}} \quad 3.18
\]

For best result, these values are not taken as the final values. The most accurate method of determining the direct axis synchronous reactances, \( X_d \) is the one, that has already been described in (A) above. The most accurate value of quadrature axis synchronous reactance, \( X_q \) can be found out by using the above information i.e \( X_{ds} \) and \( X_{d} \). Quadrature axis synchronous reactance, \( X_q = (X_{qs}/X_{ds}) \times X_{d} = (E_{min}/I_{max})(I_{min}/E_{max}) \times X_{d} \) per unit Hence the accurate value of \( X_q \) can be found out by recording minimum and maximum values of the above quantities. Accurate results can be obtained, if the oscillograms are taken during experimentation for stator current, stator voltage and injected voltage across the field. It may be noted clearly, that for synchronous machine \( X_d \) is greater than \( X_q \) i.e. \( X_d > X_q \).

Important caution for conducting slip test:-

1) Slip should be extremely low during experimentation. In case of high slip (more than about 5%) following effects may be observed: - (a) Current induced in the damper winding
of alternator will produce an appreciable error. (b) Induced voltage in the open circuit field may reach dangerous values.

2) It should be assured that the induced voltage in the open circuit is less than the rating of the voltmeter connected in the circuit.

![Fig. 3.8: Slip Test for determination of direct axis synchronous reactance and quadrature axis synchronous reactance of a salient pole synchronous machine.](image-url)
MODULE-II

CHAPTER-4

Parallel Operation of Three Phase AC Synchronous Generators
4.1 PARALLEL OPERATION OF AC GENERATORS

Why are synchronous generators operated in parallel?

There are several major advantages of parallel operation of AC generators:

a. Several AC generators can supply a bigger load than one machine by itself.

b. Having many synchronous generators increases the reliability of the power system, since the failure of anyone of them does not cause a total power loss to the load.

c. Having many synchronous generators operating in parallel allows one or more of them to be removed for shutdown and preventive maintenance.

d. If only one AC generator is used and the generator is not operating at near full load, then it will be relatively inefficient. With several smaller machines operating in parallel, it is possible to operate only a fraction of them. The machines those operate near full load are more efficiently.

![Diagram of Generators in Parallel](image)

Fig. 4.1: A generator being paralleled with a running power system.

4.2 The Conditions Required for Paralleling

Fig. 4.1 represents a synchronous generator $G_1$ supplying power to a load, with another generator $G_2$ about to be paralleled with $G_1$ by closing the switch $S_1$. There must be some definite conditions met before the switch can be closed and the two generators connected. If the switch is closed arbitrarily at any moment, the generators will be severely damaged, and the load may lose power. If the voltages are not exactly the same in each of the conductors tied together, there will be a very large amount of current flow occurs, when the switch is closed. In order to avoid this problem, each of the three phases must have exactly the same voltage magnitude and phase angle as the conductor to which it is connected. To maintain this match, the following paralleling conditions must be following:

I. The rms line voltages of the two generators must be equal.

II. The two generators must have the same phase sequence.

III. The phase angles of the two a phases must be equal.

IV. The frequency of the new generator, called the incoming generator, must be slightly higher than that of the frequency of the running system.

The condition-I is required for two sets of voltages to be identical, they must of course have the same rms magnitude of voltage. The voltage in phases a and a' will be completely identical at all times if both their magnitudes and their angles are the same, which explains condition -III. Condition -II ensures that the sequence in which the peak voltages per phase in the two generators is the same. If the phase sequence is different (as shown in Fig.4.2), then even though
one pair of voltages (the ‘a’ phases) are in phase, the other two pairs of voltages are 1200 out of phase. If the generators were connected in this manner, there would be no problem with phase ‘a’, but huge currents would likely to flow in phases ‘b’ and ‘c’, damaging both machines. To correct a phase sequence problem, simply swap the connections on any two of the three phases on one of the machines. If the frequencies of both the generators are not extremely nearly equal although they are connected together, a large power transients will occur until the generators stabilize at a common frequency. The frequencies of the two machines must be very nearly equal, but they cannot be exactly equal. They must differ by a small amount so that the phase angles of the oncoming machine will change slowly with respect to the phase angles of the running system. In that way, the angles between the voltages can be observed and switch $S_1$ can be closed when the systems are exactly in phase.

Fig. 4.2: (a) The two possible phase sequences of a three-phase system. (b) The three-light-bulb method for checking phase sequence.

4.3 The General Procedure for Paralleling Generators

Suppose that generator $G_1$ is to be connected to the running system as shown in Fig.4.2. The following steps should be taken to complete the parallel connection of the generators.

First of all, the field current of the oncoming generator should be adjusted until its terminal voltage is equal to the line voltage of the running system. The voltages are measured using voltmeters. Secondly, the phase sequence of the oncoming generator must be compared with the phase sequence of the running system. The phase sequence can be checked in a number of different ways. One way is to alternately connect a small induction motor to the terminals of each of the two generators. If the motor rotates in the same direction each time, and then the phase sequence is the same for both generators. If the motor rotates in opposite directions, then the phase sequences differ, and two of the conductors on the incoming generator must be reversed. Another way to check the phase sequence is the three-light-bulb method. In this approach, three light bulbs are connected across the open terminals of the switch connecting the generator to the system as shown in Fig. 5.2(b). As the phase changes between the two systems, the light bulbs first get bright (large phase difference) and then get dim (small phase difference).
If all three bulbs get bright and dark together, then the systems have the same phase sequence. If the bulbs brighten in succession, then the systems have the opposite phase sequence, and one of the sequences must be reversed.

Next, the frequency of the oncoming generator is adjusted to be slightly higher than the frequency of the running system. This is done first by watching a frequency meter until the frequencies are close and then by observing changes in phase between the generators. The oncoming generator is adjusted to a slightly higher frequency so that when it is connected, it will come on the line supplying power as a generator, instead of consuming it as a motor. When the frequencies are very nearly equal, the voltages in the two systems will change phase with respect to each other very slowly. The phase changes are observed, and when the phase angles are equal, the switch connecting the two systems together is shut.

How can one tell when the two systems are finally in phase? A simple way is to watch the three light bulbs described above in connection with the discussion of phase sequence. When the three light bulbs all go out, the voltage difference across them is zero and the systems are in phase. This simple scheme works, but it is not very accurate. A better approach is to employ a synchroscope. A synchroscope is a meter that measures the difference in phase angle between the phases of the two systems. The face of a synchroscope is shown in Fig. 4.3. The dial shows the phase difference between the two a phases, with 0 (meaning in phase) at the top and 1800 at the bottom. Since the frequencies of the two systems are slightly different, the phase angle on the meter changes slowly. If the oncoming

![Synchroscope](image)

Fig. 4.3 Synchroscope.

generator or system is faster than the running system, then the phase angle advances and the synchroscope needle rotates clockwise. If the oncoming machine is slower, the needle rotates counterclockwise. When the synchroscope needle is in the vertical position, the voltages are in phase, and the switch can be shut to connect the systems.

Notice, though, that a synchroscope checks the relationships on only one phase. It gives no information about phase sequence. In large generators belonging to power systems, this whole process of paralleling a new generator to the line is automated, and a computer does this job. For smaller generators, though, the operator manually goes through the paralleling steps just described.

4.4 Frequencies-Power and Voltage-Reactive Power Characteristics of a Synchronous Generator

All generators are driven by a prime mover, which is the generator's source of mechanical power. The most common type of prime mover is a steam turbine, but other types include diesel engines, gas turbines, water turbines, and even wind turbines. Regardless of the original power source, all prime movers tend to behave in a similar fashion as the power drawn from them increases, i.e., the speed at which they run tends to decrease. The decrease in speed is generally nonlinear. Therefore, different forms of governor mechanism are usually included to make the decrease in speed linear with the increase in power demand.
Whatever governor mechanism is present on a prime mover, it will always be adjusted to provide a slight drooping characteristic with increasing load. The speed droop (SD) of a prime mover is defined by the equation

\[ SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100 \]

Where \( n_{nl} \) is the no-load prime-mover speed and \( n_{fl} \) is the full-load prime-mover speed. Most generator prime movers have a speed droop of 2 to 4 percent. It has to be noted that, most governors have some type of set point adjustment to allow the no-load speed of the turbine to be varied. A typical speed-vs-power plot is shown in Fig.5.3. Since the shaft speed is related to the resulting electrical frequency by Equation (4.2) given below.

\[ f_e = \frac{n_{nl} 120}{P} \]

The electrical power output of a synchronous generator is related to its frequency. A typical plot of frequency versus power is shown in Fig. 4.4 (b). Frequency-power characteristics of this plot play an essential role in the parallel operation of synchronous generator. The relationship between frequency and power can be described quantitatively by the equation

\[ P = S_p (f_{nl} - f_{syn}) \]

Where \( P \) = power output of the generator in kW
\( f_{nl} \) = no-load frequency of the generator in Hz
\( f_{fl} \) = full-load frequency of the generator in Hz
\( f_{syn} \) = operating frequency of system in Hz
\( s_p \) = slope of curve, in kW/Hz

Fig. 4.4(a) The speed-versus-power curve for a typical prime mover. (b) The resulting frequency-versus-power curve for the generator.
A similar relationship can be derived for the reactive power $Q$ and terminal voltage $VT$. As previously seen, when a lagging load is added to a synchronous generator, its terminal voltage drops. Similarly, when a leading load is added to an asynchronous generator, its terminal voltage increases. Therefore, it is possible to make a plot of terminal voltage versus reactive power, such type of plot has a drooping characteristic like the one shown in Fig. 4.4. This characteristic is not intrinsically linear, but voltage regulators of many generators include a feature to make it so. The Q-V characteristic curve can be moved up and down by changing the no-load terminal voltage set point on the voltage regulator. As like as the frequency-power characteristic, this curve plays an important role in the parallel operation of synchronous generators.

![Fig. 4.5: The curve of terminal voltage ($V_T$) versus reactive power ($Q$) for a synchronous generator.](image)

The relationship between the terminal voltage and reactive power of a generator can be expressed by an equation similar to the frequency-power relationship [Equation(4.3)], if the voltage regulator produces an output that is linear with changes in reactive power. It is important to realize that, when a single generator is operating individually, the real power $P$ and reactive power $Q$ supplied by the generator will be the amount demanded by the load attached to the generator. The $P$ and $Q$ supplied cannot be controlled by the generator's controls. Therefore, for any given real power, the governor set point controls the generator's operating frequency $f_e$ and for any given reactive power, the field current controls the generator's terminal voltage $VT$.

In summary, for a generator is operating by itself supplying the system loads, then;

I. The real and reactive power supplied by the generator will be the amount demanded by the attached load.
II. The governor set points of the generator will control the operating frequency of the power system.
III. The field current (or the field regulator set points) controls the terminal voltage of the power system.

4.5 Operation of Generators in Parallel with Large Power Systems

When a synchronous generator is connected to a power system, the power system is often so large that nothing the operator of the generator does will have much of an effect on the power system. An example of this situation is the connection of a single generator to the U.S. power grid. The U.S. power grid is so large that no reasonable action on the part of the one generator can cause an observable change in overall grid frequency.
This idea is idealized in the concept of an infinite bus. An infinite bus is a power system so large that its voltage and frequency do not vary regardless of how much real and reactive power is drawn from or supplied to it. The power frequency characteristic of such a system is shown in Fig.4.5(a), and the reactive power-voltage characteristic is shown in Fig.4.5(b).

![Power frequency characteristic and reactive power-voltage characteristic](image)

To understand the behavior of a generator connected to such a large system, examine a system consisting of a generator and an infinite bus in parallel supplying a load. Assume that the generator's prime mover has a governor mechanism, but the field is controlled manually by a resistor. It is easier to explain synchronous generator operation without considering an automatic field current regulator, so this discussion will ignore the differences caused by the field regulator when one is present. Such a system is shown in Fig.4.6(a).

![Synchronous generator operating in parallel with an infinite bus](image)

**Fig.4.6** (a) A synchronous generator operating in parallel with an infinite bus.

(b) The frequency-versus-power diagram (or house diagram) for a synchronous generator in parallel with an infinite bus.

When a generator is connected in parallel with another generator or a large system, the frequency and terminal voltage of all the machines must be the same, since their output conductors are tied together. Therefore, their real power frequency and reactive power-voltage characteristics can be plotted back to back, with a common vertical axis. Such a sketch,
sometimes informally called a house diagram, is shown in Fig. 4.6(b). Consider that the generator has just been paralleled with the infinite bus according to the previously described procedure. Then the generator will be essentially “floating” on the line and supplying a small amount of real power and little or no reactive power. Such a situation is shown in Fig. 4.7.

![Fig. 4.7: The frequency-versus-power diagram at the moment just after paralleling.](image)

Suppose the generator had been paralleled with the line but, instead of being at a slightly higher frequency than that of the running system, it was at a slightly lower frequency. In this case, when the process of parallel connection is completed, the resulting situation can be described as shown in Fig. 4.8. This characteristic shows that the no-load frequency of the generator is less than the system's operating frequency. The power supplied by the generator at this frequency is actually negative. In the other way we can say, when the generator's no-load frequency is less than the system's operating frequency, the generator actually consumes electric power instead of consuming it that the oncoming machine's frequency is adjusted

![Fig. 4.8: The frequency-versus-power diagram if the no-load frequency of the generator were slightly less than system frequency before paralleling.](image)

higher than the running system's frequency. Many of the real generators have a provision for reverse-power trip, which is connected to them, so it is imperative that they can be paralleled with their frequency higher than that of the running system. If such a generator ever starts to consume power, it will be automatically disconnected from the line.

What happens when its governor set points are increased and the generator has been connected? The effect of this increase is to increase the no-load frequency of the generator. Since the frequency of the system is unchanged, the power supplied by the generator increases. This is shown by the house diagram in Fig. 4.9 (a) and phasor diagram in Fig. 4.9 (b). Note that, in the phasor diagram $E \sin \delta$ (which is proportional to the power supplied as long as $VT$ is constant) has increased, while the magnitude of $E_A (= k\rho \omega)$ remains constant, since both the field current $I_F$ and the speed of rotation $\omega$ are unchanged. As the governor set points are further increased, the
no-load frequency increases and the power supplied by the generator increases. Similarly, as the power output increases, $E_A$ remains at constant magnitude while $E \sin \delta$ is further increased.

What happens in this system if the power output of the generator is increased until it exceeds the power consumed by the load? If this occurs, the extra power generated flows back into the infinite bus. The infinite bus ideally can supply or consume any amount of power without a change in frequency, so the extra power will be consumed. As soon as the real power of the generator has been adjusted to the desired value, the phasor diagram of the generator looks like Fig 4.9 (b). It can be observed that, at this time the generator is actually operating at a slightly leading power factor, supplying negative reactive power. In other words, the generator can be said to be consuming reactive power.

How can the generator be adjusted so that it will supply some reactive power $Q$ to the system? This can be done by adjusting the field current of the machine. To understand why this is true, it becomes necessary to consider the constraints on the generator's operation under these circumstances. The first constraint on the generator is that the power must remain constant when $I_F$ is changed. The power into a generator is given by the equation $P_{in} = T_{ind} \omega_m$. Now, the prime mover of an alternator has a fixed torque-speed characteristic for any given governor setting. This curve changes only when the governor set points are changed. Since the generator is tied to an infinite bus, its speed cannot change. If the generator's speed does not change and the governor set points have not been changed, the power supplied by the generator must remain constant.

![Diagram](image)

Fig 4.9 Effect of increasing the governor's set points on (a) the house diagram (b) the phasor diagram

If the power supplied is constant as the field current is changed, then the distances proportional to the power in the phasor diagram ($I_A \cos(\theta)$ and $E_A \sin \delta$) cannot change. When the field current is increased, the flux $\Phi$ increases, and therefore $E_A$ ($= K \Phi \omega$) increases. If $E_A$ increases, but $E_A \sin \delta$ must remain constant, then the phasor $E_A$ must "slide" along the line of constant power, as shown in Fig. 4.10. Since $V_T$ is constant, the angle of $jXsI_A$ changes as shown, and therefore the angle and magnitude of $I_A$ changes. Notice that as a result the distance proportional to $Q = I_A \sin$...
(θ) increases. In other words, increasing the field current in a synchronous generator operating in parallel with an infinite bus increases the reactive power output of the generator.

![Fig. 4.10: The effect of increasing the generator's field current on the phasor diagram of the machine.](image)

To summarize, when a generator is operating in parallel with an infinite bus:

1. The frequency and terminal voltage of the synchronous generator are controlled by the system to which it is connected.
2. The governor set points of the synchronous generator control the real power supplied by the generator to the system.
3. The field current in the synchronous generator controls the reactive power supplied by the generator to the system.

4.6 Operation of Generators in Parallel with Other Generators of the Same Size

When a single synchronous generator operated individually, the real and reactive powers (P and Q) supplied by the generator were fixed and constrained to be equal to the power demanded by the load. Also, the frequency and terminal voltage were varied by the governor set points and the field current. When a synchronous generator operated in parallel with an infinite bus, the frequency and terminal voltage were necessarily constant by the infinite bus, and the real and reactive powers were varied by the governor set points and the field current.

What will happen when a synchronous generator is connected in parallel not with an infinite bus, but with another generator of the same size? What will be the effect of changing governor set points and field currents?

If a generator is connected in parallel with another one of the same size, the resulting system is as shown in Fig. 4.11(a). In this system, the basic constraint is that the sum of the real and reactive powers supplied by both the two generators must equal the P and Q demanded by the load. The system frequency is not constrained to be constant, and neither is the power of a given generator constrained to be constant. The power-frequency diagram for such type of system immediately after G2 has been paralleled to the line is shown in Fig. 4.11(b). Here, the total power $P_{TOT}$ is equal to $P_{LOAD}$ is given by

$$P_{TOT} = P_{G1} + P_{G2} - - - - - - - - - - 4.4$$

And the total reactive power $Q_{TOT}$ is given by
\[ Q_{TOT} = Q_{G1} + Q_{G2} - - - - - - 4.5 \]

What happens if the governor set points of \( G2 \) are increased? When the governor set points of \( G2 \) are increased, the power-frequency curve of \( G2 \) shifts upward, as shown in Fig. 4.11(c). The total power supplied to the load must not change. At the original frequency \( f_1 \), the power supplied by \( G1 \) and \( G2 \) will now be larger than the load demand, so the system cannot continue to operate with the same frequency as earlier. There is only one frequency at which the sum of the powers out of the two generators is equal to \( P_{LOAD} \). That frequency \( f_2 \) is higher than the original system operating frequency. At this frequency, \( G2 \) supplies more power and \( G1 \) supplies less power than before.

Therefore, when two synchronous generators are operating together, an increase in governor set points on one of them will

1. Increases the system frequency.
2. Increases the power supplied by that generator, while reducing the power supplied by the other one.

What happens if the field current of \( G2 \) is increased? The resulting behavior is analogous to the real-power situation and is shown in Fig. 4.11(d). When two generators are operating together and the field current of \( G2 \) is increased,

1. The system terminal voltage is increased.
2. The reactive power \( Q \) supplied by that generator is increased, while the reactive power supplied by the other generator is decreased.

When two generators of similar size are operating in parallel, a change in the governor set points of one of them changes both the system frequency and the power sharing between them. It would normally be desired to adjust only one of these quantities at a time. How can the power sharing of the power system be adjusted independently of the system frequency, and vice versa?

The answer is very simple. An increase in governor set points on one generator increases that machine's power and increases system frequency. A decrease in governor set points on the other generator decreases that machine's power and decreases the system frequency. Therefore, to adjust power sharing without changing the system frequency, increase the governor set points of one generator and simultaneously decrease the governor set points of the other generator (see Fig. 4.12(a)). Similarly, to adjust the system frequency without changing the power sharing, simultaneously increase or decrease both governor set points (see Fig. 4.12(b)).

When two generators of almost similar size are operating in parallel, a change in the governor set points of one of them changes both the system frequency and the power sharing between them. It would be desired to adjust only one of these quantities at a time.

How the power sharing of the power system can be adjusted independently from the system frequency, and vice versa? An increase in governor set points of one generator increases that machine's power and increases system frequency. A decrease in governor set points on the other generator decreases that machine's power and decreases the system frequency. Therefore, to adjust power sharing without changing the system frequency, increase the governor set points of one generator and simultaneously decrease the governor set points of the other generator (see Fig. 4.12(a)). Similarly, to adjust the system frequency without changing the power sharing, simultaneously increase or decrease both governor set points (see Fig. 4.12(b)).
Fig. 4.11 (a) A generator connected in parallel with another machine of the same size. (b) The corresponding house diagram at the moment generator 2 is paralleled with the system. (c) The effect of increasing generator 2's governor set points on the operation of the system. (d) The effect of increasing generator 2's field current on the operation of the system.

The answer is very simple. An increase in governor set points on one generator increases that machine's power and increases system frequency. A decrease in governor set points on the other generator decreases that machine's power and decreases the system frequency. Therefore, to adjust power sharing without changing the system frequency, increase the governor set points of one generator and simultaneously decrease the governor set points of the other generator (see Fig. 4.12(a)). Similarly, to adjust the system frequency without changing the power sharing, simultaneously increase or decrease both governor set points (see Fig. 4.12(b)).

Reactive power and terminal voltage adjustments work in an analogous fashion. To shift the reactive power sharing without changing $V_T$ simultaneously increase the field current on one generator and decrease the field current on the other (see Fig. 4.12(c)). To change the terminal
voltage without affecting the reactive power sharing, simultaneously increase or decrease both field currents (see Fig. 4.12(d)).

To summarize, in the case of two generators operating together:
1. The system is constrained in that the total power supplied by the two generators together must equal the amount consumed by the load. Neither $f_{syn}$ nor $V_T$ is constrained to be constant.
2. To adjust the real powers sharing between generators without changing $f_{syn}$ simultaneously increase the governor set points on one generator while decreasing the governor set points on the other. The machine whose governor set point was increased will assume more of the load.
3. To adjust $f_{syn}$ without changing the real power sharing, simultaneously increase or decrease both generators' governor set points.
4. To adjust the reactive power sharing between generators without changing $V_T$, simultaneously increase the field current on one generator while decreasing the field current on the other. The machine whose field current was increased will assume more of the reactive load.
5. To adjust $V_T$ without changing the reactive power sharing, simultaneously increase or decrease both generators' field currents.

It is very important that any synchronous generator intended to operate in parallel with other machines have a drooping frequency-power characteristic. If two generators have flat or nearly flat characteristics, then the power sharing between them can vary widely with only the tiniest changes in no-load speed. This problem is illustrated by Fig. 4.13. Notice that even very tiny changes in $f_{in}$ in one of the generators would cause wild shifts in power sharing. To ensure good control of power sharing between generators, they should have speed droops in the range of 2 to 5 percent.

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**Fig. 4.12 (a)** Shifting power sharing without affecting system frequency. **(b)** Shifting system frequency without affecting power sharing.
Fig. 4.12(c) Shifting reactive power sharing without affecting terminal voltage. (d) Shifting terminal voltage without affecting reactive power sharing.

Fig. 4.13: Two synchronous generators with flat frequency-power characteristics. A very tiny change in the no load frequency of either of these machines could cause huge shifts in the power sharing.
CHAPTER-5

Three Phase Synchronous Motor
5.1. Introduction:

In order to understand the principle of operation of a synchronous motor, let us examine what happens if we connect the armature winding (laid out in the stator) of a 3-phase synchronous machine to a suitable balanced 3-phase source and the field winding to a D.C source of appropriate voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. (for convenience let us assume a salient pole rotor, as shown in Fig. 50). On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e at any location in the stator there will be a North pole at some instant of time and it will become a South pole after a time period corresponding to half a cycle. (After a time = \( \frac{1}{2f} \) where \( f \) = frequency of the supply). Let us assume that the stationary South Pole in the rotor is aligned with the North Pole in the stator moving in clockwise direction at a particular instant of time, as shown in Fig. 5.1. These two poles get attracted and try to maintain this alignment (as per lenz’s law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However the rotor can not move instantaneously due to its mechanical inertia, and so it needs some time to move. In the meantime, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South Pole. So the force of attraction will no longer be present and instead the like poles experience a force of repulsion as shown in Fig. 5.2. In other words, the conditions are now suitable for the production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole would again change to North Pole after a time of \( \frac{1}{2f} \). Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to \( \frac{1}{2f} \) seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.
Fig. 5.2: Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction.

On the contrary if the rotor is brought to near synchronous speed by some external means say a small motor (known as pony motor-which could be a D.C or AC induction rotor) mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the pony motor is disconnected.

Thus the synchronous rotor cannot start rotating on its own or usually we say that the synchronous rotor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to operate at its synchronous speed. Some of the commonly used methods for starting synchronous rotor are described in the following section.

5.2: Methods of starting synchronous motor:

Basically there are three methods that are used to start a synchronous motor:

- To reduce the speed of the rotating magnetic field of the stator to a low enough value that the rotor can easily accelerate and lock in with it during one half-cycle of the rotating magnetic field's rotation. This is done by reducing the frequency of the applied electric power. This method is usually followed in the case of inverter-fed synchronous motor operating under variable speed drive applications.

- To use an external prime mover to accelerate the rotor of synchronous motor near to its synchronous speed and then supply the rotor as well as stator. Of course care should be taken to ensure that the directions of rotation of the rotor as well as that of the rotating magnetic field of the stator are the same. This method is usually followed in the laboratory- the synchronous machine is started as a generator and is then connected to the supply mains by following the
synchronization or paralleling procedure. Then the power supply to the prime mover is disconnected so that the synchronous machine will continue to operate as a motor.

• To use damper windings or amortisseur windings if these are provided in the machine. The damper windings or amortisseur windings are provided in most of the large synchronous motors in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load.

Each of these methods of starting a synchronous motor are described below in detail.

5.2.1 Motor Starting by reducing the supply Frequency

If the rotating magnetic field of the stator in a synchronous motor rotates at a low enough speed, there will be no problem for the rotor to accelerate and to lock in with the stator’s magnetic field. The speed of the stator magnetic field can then be increased to its rated operating speed by gradually increasing the supply frequency ‘f’ up to its normal 50- or 60-Hz value.

This approach to starting of synchronous motors makes a lot of sense, but there is a big problem: Where from can we get the variable frequency supply? The usual power supply systems generally regulate the frequency to be 50 or 60 Hz as the case may be. However, variable-frequency voltage source can be obtained from a dedicated generator only in the olden days and such a situation was obviously impractical except for very unusual or special drive applications. But the present day solid state power converters offer an easy solution to this. We now have the rectifier-inverter and cycloconverters, which can be used to convert a constant frequency AC supply to a variable frequency AC supply. With the development of such modern solid-state variable-frequency drive packages, it is thus possible to continuously control the frequency of the supply connected to the synchronous motor all the way from a fraction of a hertz up to and even above the normal rated frequency. If such a variable-frequency drive unit is included in a motor-control circuit to achieve speed control, then starting the synchronous motor is very easy—simply adjust the frequency to a very low value for starting, and then raises it up to the desired operating frequency for normal running. When a synchronous motor is operated at a speed lower than the rated speed, its internal generated voltage (usually called the counter EMF) $E= Kf \omega$ will be smaller than normal. As such the terminal voltage applied to the motor must be reduced proportionally with the frequency in order to keep the stator current within the rated value. Generally, the voltage in any variable-frequency power supply varies roughly linearly with the output frequency.

5.2.2 Motor Starting with an External Motor:

The second method of starting a synchronous motor is to attach an external starting motor (pony motor) to it and bring the synchronous machine to near about its rated speed (but not exactly equal to it, as the synchronization process may fail to indicate the point of closure of the main switch connecting the synchronous machine to the supply system) with the pony motor. Then the output of the synchronous machine can be synchronised or paralleled with its power supply system as a generator, and the pony motor can be detached from the shaft of the machine or the supply to the pony motor can be disconnected. Once the pony motor is turned OFF, the shaft of the machine slows down, the speed of the rotor magnetic field $B_R$ falls behind $B_{net}$ momentarily and the synchronous machine continues to operate as a motor. As soon as it begins to operate as a motor the synchronous motor can be loaded in the usual manner just like any motor.
This whole procedure is not as cumbersome as it sounds, since many synchronous motors are parts of motor-generator sets, and the synchronous machine in the motor-generator set may be started with the other machine serving as the starting motor. Moreover, the starting motor is required to overcome only the mechanical inertia of the synchronous machine without any mechanical load (load is attached only after the synchronous machine is paralleled to the power supply system). Since only the motor’s inertia must be overcome, the starting motor can have a much smaller rating than the synchronous motor it is going to start. Generally most of the large synchronous motors have brushless excitation systems mounted on their shafts. It is then possible to use these exciters as the starting motors. For many medium-size to large synchronous motors, an external starting motor or starting by using the exciter may be the only possible solution, because the power systems they are tied to may not be able to handle the starting currents needed to use the damper (amortisseur) winding approach described next.

5.2.3 Motor Starting by Using damper (Amortisseur) Winding:
As already mentioned earlier most of the large synchronous motors are provided with damper windings, in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load. Damper windings are special bars laid into slots cut in the pole face of a synchronous machine and then shorted out on each end by a large shorting ring, similar to the squirrel cage rotor bars. A pole face with a set of damper windings is shown in Fig.. When the stator of such a synchronous machine is connected to the 3-Phase AC supply, the machine starts as a 3-Phase induction machine due to the presence of the damper bars, just like a squirrel cage induction motor. Just as in the case of a 3-Phase squirrel cage induction motor, the applied voltage must be suitably reduced so as to limit the starting current to the safe rated value. Once the motor picks up to a speed near about its synchronous speed, the DC supply to its field winding is connected and the synchronous motor pulls into step i.e. it continues to operate as a Synchronous motor running at its synchronous speed.

5.3: Behaviour of a synchronous motor
The behaviour of a synchronous motor can be predicted by considering its equivalent circuit on similar lines to that of a synchronous generator as described below.

5.3.1 Equivalent circuit model and phasor diagram of a synchronous motor:
The equivalent-circuit model for one armature phase of a cylindrical rotor three phase synchronous motor is shown in Fig. 5.3 exactly similar to that of a synchronous generator except that the current flows in to the armature from the supply. All values are given per phase. Applying Kirchhoff’s voltage law to Fig. 5.3,

\[ V_T = I_a R_a + jI_a (X_l + X_{as}) + E_f \]  \hspace{2cm} 5.1

Combining reactances, we have,
\[ X_s = X_l + X_{as} \]  \hspace{2cm} 5.2

Substituting Eqn. 5.2 in Eqn. 5.1
\[ V_T = I_a (R_a + jX_s) + E_f \]  \hspace{2cm} 5.3

\[ V_T = I_a Z_s + E_f \]  \hspace{2cm} 5.4

Where:
\( R_s \) = armature resistance (\( \Omega \)/phase)
\( X_l \) = armature leakage reactance (\( \Omega \)/phase)
\( X_s \) = synchronous reactance (\( \Omega \)/phase)
Zs = synchronous impedance (Ω/phase)
VT = applied voltage/phase (V)
Ia = armature current/phase (A)

The phasor diagram shown in Fig. 5.4, illustrates the method of determining the counter EMF which is obtained from the phasor equation:

\[ E_f = V_T - I_a Z_s \]  \hspace{1cm} (5.5)

The phase angle \( \delta \) between the terminal voltage VT and the excitation voltage E in Fig. 5.4 is usually termed the torque angle. The torque angle is also called the load angle or power angle.

5.3.2 Synchronous-motor power equation:
Except for very small machines, the armature resistance of a synchronous motor is relatively insignificant compared to its synchronous reactance, so that Eqn. 5.4 to be approximated to

\[ V_T = I_a j X_s + E_f \]  

5.6

The equivalent-circuit and phasor diagram corresponding to this relation are shown in Fig. 5.5(a) and Fig. 5.5(b). These are normally used for analysing the behaviour of a synchronous motor, due to changes in load and/or changes in field excitation. From this phasor diagram, we have,

\[ I_a X_s \cos \theta_i = -E_f \sin \delta \]  

5.7

Multiplying through by VT and rearranging terms we have,

\[ I_a \cos \theta_i = \frac{-V_T E_f}{X_s} \sin \delta \]  

5.8

Since the left side of Eqn. 5.8 is an expression for active power input and as the winding resistance is assumed to be negligible this power input will also represent the electromagnetic power developed, per phase, by the synchronous motor. Thus,

\[ P_{in/\text{ph}} = V_T I_a \cos \theta_i \]  

5.9

Or,

\[ P_{in,\text{ph}} = \frac{-V_T E_f}{X_s} \sin \delta \]  

5.10

\[ P_{in,\text{ph}} = 3 * \frac{-V_T E_f}{X_s} \sin \delta \]  

5.11

Thus, for a three-phase synchronous motor,

\[ P_{in} = 3V_T I_a \cos \theta_i \]  

5.12

Eqn. 5.10, called the synchronous-machine power equation, expresses the electromagnetic power developed per phase by a cylindrical-rotor motor, in terms of its excitation voltage and power angle. Assuming a constant source voltage and constant supply frequency, Eqn. 5.9 and Eqn. 5.10 may be expressed as proportionality that are very useful for analyzing the behaviour of a synchronous-motor:

\[ P \alpha I_a \cos \theta_i \]  

5.13

\[ P \alpha E_f \sin \delta \]  

5.14
5.3.3 Effect of changes in load on armature current, power angle, and power factor of synchronous motor

The effects of changes in mechanical or shaft load on armature current, power angle, and power factor can be seen from the phasor diagram shown in Fig. 5.56: As already stated, the applied stator voltage, frequency, and field excitation are assumed constant. The initial load conditions are represented by the thick lines. The effect of increasing the shaft load to twice its initial value are represented by the light lines indicating the new steady state conditions. These are drawn in accordance with Eqn. 69 and Eqn. 70, when the shaft load is doubled both $Ia \cos \delta$ and $Ef \sin \delta$ are doubled. While redrawing the phasor diagrams to show new steady-state conditions, the line of action of the new $jIaX_s$ phasor must be perpendicular to the new $I_a$ phasor. Furthermore, as shown in Fig. 5.66, if the excitation is not changed, increasing the shaft load causes the locus of the $E_f$ phasor to follow a circular arc, thereby increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in $\phi$, resulting in an increase in power factor. As additional load is placed on the machine, the rotor continues to increase its angle of lag relative to the rotating magnetic field, thereby increasing both the angle of lag of the counter EMF phasor and the magnitude of the stator current. It is interesting to note that during all this load variation, however, except for the duration of transient conditions whereby the rotor assumes a new position in relation to the rotating magnetic field, the average speed of the machine does not change. As the load is being increased, a final point is reached at which a further increase in $\delta$ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. In fact as stated earlier, the rotor poles at this point, will fall behind the stator poles such that they now come under the influence of like poles and the force of attraction no longer exists. Thus, the point of maximum torque occurs at a power angle of approximately $90^0$ for a cylindrical-rotor machine, as is indicated by Eqn. 5.12. This maximum value of torque that causes a synchronous motor to pull out of synchronism is called the pull-out torque. In actual practice, the motor will never be operated at power angles close to $90^0$ as armature current will be many times its rated value at this load.
Fig. 5.6: Phasor diagram showing effect of changes in shaft load on armature current, power angle and power factor of a synchronous motor.

5.3.4 Effect of changes in field excitation on synchronous motor performance:

Intuitively we can expect that increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating magnetic poles of the stator. This will obviously result in a smaller power angle. This fact can also be seen in Eqn. 5.11 When the shaft load is assumed to be constant, the steady-state value of $E_f \sin \delta$ must also be constant. An increase in $E_f$ will cause a transient increase in $E_f \sin \delta$, and the rotor will accelerate. As the rotor changes its angular position, $\delta$ decreases until $E_f \sin \delta$ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed, as it should run only at the synchronous speed. This change in angular position of the rotor magnets relative to the poles of rotating magnetic field of the stator occurs in a fraction of a second. The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a constant shaft load, from a constant voltage, constant frequency supply, is illustrated in Fig. 5.7.

From Eqn. 5.13, we have for a constant shaft load,

$$E_f \sin \delta = E_f \sin \delta = E_f \sin \delta = E_f \sin \delta$$  \hspace{1cm} 5.15

This is shown in Fig. 5.7, where the locus of the tip of the $E_f$ phasor is a straight line parallel to the VT phasor. Similarly, from Eqn. 5.13 for a constant shaft load,

$$I_{a1} \cos \phi_{i1} = I_{a2} \cos \phi_{i2} = I_{a3} \cos \phi_{i3} = I_a \cos \phi_i$$  \hspace{1cm} 5.16
This is also shown in Fig. 5.7, where the locus of the tip of the Iaphasor is a line perpendicular to the VT phasor.

Note that increasing the excitation from Ef1 to Ef3 in Fig. 5.7 caused the phase angle of the current phasor with respect to the terminal voltage V (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called normal excitation. Excitation greater than normal is called over excitation, and excitation less than normal is called under excitation. Furthermore, as indicated in Fig. 5.7, when operating in the overexcited mode, \(|Ef| > |VT|\). In fact a synchronous motor operating under over excitation condition is sometimes called a synchronous condenser.

5.3.5 V curves

Curves of armature current vs. field current (or excitation voltage to a different scale) are called V curves, and are shown in Fig. 58 for typical values of synchronous motor loads. The curves are related to the phasor diagram in Fig. 5.7, and illustrate the effect of the variation of field excitation on armature current and power factor for typical shaft loads. It can be easily noted from these curves that an increase in shaft loads require an increase in field excitation in order to maintain the power factor at unity.
The locus of the left most point of the V curves in Fig. 5.8 represents the stability limit ($\delta=\pm 90^\circ$). Any reduction in excitation below the stability limit for a particular load will cause the rotor to pullout of synchronism.

The V curves shown in Fig. 5.8 can be determined experimentally in the laboratory by varying $I_f$ at constant shaft load and noting $I_a$ as $I_f$ is varied. Alternatively the V curves shown in Fig. 5.8 can be determined graphically by plotting $|I_a|$ vs. $|E_f|$ from a family of phasor diagrams as shown in Fig. 5.7, or from the following mathematical expression for the V curves

\[
(I_aX_s)^2 = V_f^2 + E_f^2 - 2V_fE_f \cos \delta
\]

\[
= V_f^2 + E_f^2 - 2V_fE_f \sqrt{1 - \sin^2 \delta}
\]

\[
= V_f^2 + E_f^2 - 2 \sqrt{V_f^2E_f^2 - V_f^2E_f^2\sin^2 \delta}
\]

\[
I_a = \frac{1}{X_s} \sqrt{V_f^2 + E_f^2 - 2 \sqrt{V_f^2E_f^2 - X_s^2P_{in.ph}^2}}
\]

Eqn. 5.17 is based on the phasor diagram and the assumption that $R_a$ is negligible. It is to be noted that instability will occur, if the developed torque is less than the shaft load plus friction and windage losses, and the expression under the square root sign will be negative.

The family of V curves shown in Fig. 5.8 represent computer plots of Eqn. 5.17, by taking the data pertaining to a three-phase 10 hp synchronous motor i.e $V_{ph}= 230V$ and $X_s= 1.2\Omega$/phase.

Fig. 5.8: Family of representative V curves for a synchronous motor
5.3.6 Synchronous-motor losses and efficiency:

The flow of power through a synchronous motor, from stator to rotor and then to shaft output, is shown in Fig. 5.9. As indicated in the power-flow diagram, the total power loss for the motor is given by

\[ P_{loss} = P_{scl} + P_{core} + P_{fcl} + P_{f,w} + P_{stray} \text{ W} \quad 5.18 \]

where:

- \( P_{scl} \) = stator-copper loss
- \( P_{fcl} \) = field-copper loss
- \( P_{core} \) = core loss
- \( P_{f,w} \) = friction and windage loss
- \( P_{stray} \) = stray load loss

Except for the transient conditions that occur when the field current is increased or decreased (magnetic energy stored or released), the total energy supplied to the field coils is constant and all of it is consumed as \( I^2R \) losses in the field winding. Just as in the case of the synchronous generator, the overall efficiency of a synchronous motor is given by

\[ \eta = \frac{P_{shaft}}{P_{in} + P_{field}} = \frac{P_{shaft}}{P_{shaft} + P_{loss}} \quad 5.19 \]

Generally, the nameplates of synchronous motors and manufacturers’ specification sheets customarily provide the overall efficiency for rated load and few load conditions only. Hence, only the total losses at these loads can be determined. The separation of losses into the components listed in Eqn. 5.18 needs a very involved test procedure in the laboratory. However, a closer approximation of the mechanical power developed can be calculated by subtracting the copper losses of the armature and field winding if these losses can be calculated. The shaft power can then be calculated subtracting the mechanical losses from the mechanical power developed.

![Fig. 5.9: Power flow diagram for a synchronous motor](image)

- \( v \) = velocity of the bar relative to the magnetic field
- \( B \) = magnetic flux density vector
- \( I \) = length of conductor in the magnetic field
MODULE-III

CHAPTER-6

Three Phase Transformer
6.1 Objective:
After studying this unit, the students will be able to
• Identify the common types of transformers from their schematic diagrams.
• Choose the correct transformer kVA for the application, given the voltage, current, and phase requirement of a load
• Point out the advantages and disadvantage of connecting a bank of 3-phase transformer.
• Connect two transformers in parallel properly.
• Convert a balance 3-phase to balance 2-phase and vice versa with suitable transformation connections.

6.2 Introduction:
Modern electrical systems are almost exclusively three-phase systems, which has been adopted world over to generate, transmit and distribute electrical power. Therefore to change the level of voltages in the system three phase transformers should be used.

When two poly phase systems have different voltages and/or phase angles, these systems can be interconnected using transformers having various possible types of connections. Any one of these connections can be accomplished either with a bank of single-phase transformers or by a single poly phase transformer.

A poly-phase two-winding transformer contains a number of sets of primary and secondary windings. Each set wound around a separate magnetic core leg. A three-phase two-winding transformer has three sets of primary and secondary windings, and a two-phase two-winding transformer has two sets of primary and secondary windings.

In Electrical Machine-I the basic theory of operation of a two-winding transformer and the transformer laws were discussed. This chapter focuses, the principles of the two-winding transformer are applied to poly phase systems.

6.3 Construction:
A three-phase transformer can be constructed by having three primary and three secondary windings on a common magnetic circuit as shown in Fig. 6.1(i). The primaries as well as secondary may be connected in star or delta.

Here three, single phase transformers are so placed that they share a common central limb. The primary and the secondary windings of each phase are placed on their outer limbs. If the primary windings are connected to a balanced 3-phase supply, the fluxes $\phi_a(t)$, $\phi_b(t)$ and
$\phi_c(t)$ will be produced in the cores differing in time phase mutually by $120^\circ$. The return paths of these fluxes are through the central limb of the core structure. In other words the central limb carries sum of these three fluxes. Since instantaneous sum of the fluxes, $\phi_A(t) + \phi_B(t) + \phi_C(t) = 0$, no flux lines will exist in the central limb at any time. As such the central limb can be removed without affecting the working of the transformer; this modification gives a three leg core type 3-phase transformer as shown in fig 6.1(ii).

![Fig. 6.1 Three phase core type transformer](image)

(i) (ii)

In this case, any two legs will act as a return path for the flux in the third leg. For example, if flux is $\phi$ in one leg at some instant, then flux are $\phi/2$ in the opposite direction through the other two legs at the same instant.

A further simplification of the structure can be obtained by bringing the limbs in the same plane as shown in the figure 6.2. In core structure of figure 6.1, we note that the reluctance seen by the three fluxes are same, Hence magnetizing current will be equal in all the three phases. In the simplified core structure of figure 6.2, reluctance encountered by the flux $\phi_b(t)$ is different from the reluctance encountered by fluxes $\phi_A(t)$ and $\phi_C(t)$, hence the magnetizing currents or the no load currents drawn will remain slightly unbalanced. This degree of unbalanced for no load current has practically no influence on the performance of the loaded transformer. Transformer having this type of core structure is called the core type transformer.

![Fig. 6.2 Simplified three phase three limb core type transformer](image)

A 3-phase shell type transformer can be obtain by placing three single phase transformers side by side as shown in fig. 6.3. In the magnetic circuits 2 and 3 the net flux is found to be the resultant
of half of the flux produce by two phases at any instant, which is 86.6% of the flux produce by one phase. In the central area the flux is \( \phi = \phi_A = \phi_B = \phi_C \). The flux carried by the magnetic circuits 1 and 4 is \( \frac{\phi}{2} \), therefore the areas are 50% of the central limb, where as the cross-sectional area of the circuit 2 and 3 are 86.6% of the central limb.

If the winding of phase B is wound in the reversed direction which is shown by the dot mark as shown in fig. 6.3(ii), the direction of phase B flux in the limb 2 and 3 reverses, for which the resultant flux in these limbs becomes \( \frac{\phi}{2} \), thus the area of cross-section of the portions marked 1, 2, 3 and 4 is equal to one half of the central limb area. By this area the weight of the core material can be reduced considerably.

![Fig. 6.3 Three phase shell type transformer (i) with 3 windings wound in same direction, (ii) with central winding wound in the reverse direction.](image)

For the same capacity, a 3-phase transformer weighs less, occupies less space and the cost is much less compared to a bank of single phase transformers. For these advantages of a single unit of 3-phase transformer, In fact all large capacity transformers are a single unit of three phase transformer.

A disadvantage of the three-phase transformer is when one phase becomes defective; the entire three-phase unit must be removed from service. Where as in other type case the other two transformers may be reconnected to supply service on an emergency basis until repairs can be made.

**6.4 Difference between a single unit 3 phase transformer and bank of 3 single phase transformers**

The choice of transformer between the two can be done on the basis of the advantages and disadvantages listed below it the tabular form
### Bank of 3 single phase transformers vs Single unit 3 phase transformer

<table>
<thead>
<tr>
<th></th>
<th>Bank of 3 single phase transformers</th>
<th>Single unit 3 phase transformer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>More costly due to more iron, three separate tanks, more oil and more auxiliary equipments.</td>
<td>Less costly as less volume of iron, one tank so less oil and less auxiliary equipments.</td>
</tr>
<tr>
<td>2</td>
<td>As three separate transformers it requires six h.v. bushings which increase the cost.</td>
<td>It requires only three bushings which reduces the cost.</td>
</tr>
<tr>
<td>3</td>
<td>More floor space is required which increases the capital.</td>
<td>Less floor space is required which decreases the capital.</td>
</tr>
<tr>
<td>4</td>
<td>If one phase/transformer damaged, then it can be easily replaced by a single phase unit.</td>
<td>If one phase is damaged, whole transformer has to be replaced, which increases the cost.</td>
</tr>
<tr>
<td>5</td>
<td>Only one single phase transformer is required as standby unit.</td>
<td>One complete three phase transformer is required as standby unit.</td>
</tr>
<tr>
<td>6</td>
<td>Because of more iron part, more iron loss hence less efficient.</td>
<td>Comparatively less iron part so more efficient.</td>
</tr>
<tr>
<td>7</td>
<td>Easy for transportation as each unit is small.</td>
<td>Comparatively to the first case more difficult for transportation.</td>
</tr>
</tbody>
</table>

### 6.5 Three-Phase Transformer connections

A three-phase transformer can be built by suitably connecting a bank of three single phase transformers or by one three-phase transformer. The primary or secondary windings may be connected in either star (Y) or delta (Δ) arrangement. The four most common connections are (i) Y-Y (ii) Δ - Δ (iii) Y- Δ and (iv) Δ - Y. These four connections are described below. In the figure, the windings at the left are the primaries and those at the right are the secondaries.

The primary and secondary voltages and currents are also shown. The notations used are given by:

\[
V_1, V_2: \text{Rated primary and secondary phase voltages},
\]

\[
I_1, I_2: \text{Rated primary and secondary phase currents},
\]

\[
N_1, N_2: \text{Primary and secondary number of turns},
\]

\[
S= V_1 I_1= V_2 I_2=\text{Rated kVA}
\]

#### 6.5.1 Star-Star (Y-Y) Connection

To apply a rated voltage to the primary terminals a line to line voltage of \(\sqrt{3}V_1\) is supplied, so that the primary rated voltage \(V_1\) is impressed across each of the primary coils of the individual transformer. This ensures \(V_2\) to be induced across each of the secondary coil and the line to line voltage in the secondary will be \(\sqrt{3}V_2\). Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. From the individual rating of each transformer, we know maximum allowable currents of primary and secondary windings are \(I_1\) and \(I_2\) respectively. Since
secondary side is connected in star, line current and the winding currents (phase current) are same. Therefore total kVA that can be supplied to a balanced 3-phase load is \( \sqrt{3} (V_p)(I_k) = \sqrt{3} (\sqrt{3} V_2) I_2 = 3V_2I_2 = 3s \) i.e. three times the Kva rating of each single phase transformer.

Phase voltage transformation ratio = \( \frac{V_2}{V_1} = \frac{N_2}{N_1} = k \)

Line voltage transformation ratio = \( \frac{\sqrt{3}V_2}{\sqrt{3}V_1} = \frac{N_2}{N_1} = k \)

Note:
1. Unless and until mention simply turns ratio in the sense phase turns ratio and transformation ratio sense phase transformation ratio.
2. The 3-phase transformers are always specified with line voltages.

This type of connection requires less insulation as the phase voltage is less than the line voltage. For which it require less number of turns/phase but more cross sectional area of conductor. It is economical for high voltage low current rating transformers. As the cross sectional areas of conductors are more it is more mechanically strong.

![Star-Star connections](image)

**Fig. 6.3 Star-Star connections**

With both primary and secondary connected in star no closed path exists among the windings. As the triplen harmonics are always in phase, by virtue of the Y connection they get canceled in the line voltages. Non-triplen harmonics like fundamental become 3 times phase value and appear in the line voltages. Line currents remain sinusoidal except for non-triplen harmonic currents.

### 6.5.2 Delta- Delta (△-△) Connection

As discussed above, to apply a rated voltage to the primary terminals a line to line voltage of \( V_1 \) is supplied, so that the primary rated voltage \( V_1 \) is impressed across each of the primary coils of the individual transformer, as for delta connection both line and phase voltages
are same. This ensures $V_2$ to be induced across each of the secondary coil and the line to line voltage in the secondary will also be $V_2$. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. Since secondary side is connected in delta, we can connect a load in such a way that the winding currents (phase current) should not exceed $I_2$, for which the line current becomes $\sqrt{3}I_2$. When the current in secondary winding becomes $I_2$, the corresponding reflected phase current in the primary becomes $I_1$ and the line current becomes $\sqrt{3}I_1$. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_n)(I_k) = \sqrt{3}V_2(\sqrt{3}I_2) = 3V_2I_2 = 3s$ i.e. three times the Kva rating of each single phase transformer.

Phase voltage transformation ratio= Line voltage transformation ratio $= \frac{V_2}{V_1} = \frac{N_2}{N_1} = k$

As the phase and line voltages are same it requires more insulation but more number of turns/phase, where as the cross sectional areas of conductors are less compare to $Y-Y$ connection. For which it is more economical for low voltage and high current applications. Due to the less cross sectional areas of conductors it is mechanically weak compare to $Y-Y$ transformers.

![Fig. 6.4 Delta-Delta connections](image)

With mesh connection on both primary side and secondary side a closed path is available for the triplen harmonics to circulate currents. Thus the supply current is nearly sinusoidal (but for the non-triplen harmonic currents). The triplen harmonic currents inside the closed mesh winding correct the flux density wave to be nearly sinusoidal. The secondary voltages will be nearly sinusoidal. Third harmonics currents flow both in the primary and the secondary and hence the magnitudes of these currents, so also the drops due to them will be lower.

### 6.5.3 Star-Delta ($Y$-$\Delta$) Connection
For this connection a line to line voltage of $\sqrt{3}V_1$ is supplied, so that the primary rated voltage $V_1$ is impressed across each of the primary coils of the individual transformer. This ensures $V_2$ to be induced across each of the secondary coil and the line to line voltage in the secondary will also be $V_2$ as it is connected in delta. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. Since secondary side is connected in delta, we can connect a load in such a way that the winding currents (phase current) should not exceed $I_2$, for which the line current becomes $\sqrt{3}I_2$. When the current in secondary winding becomes $I_2$, the corresponding reflected phase current in the primary becomes $I_1$ and the line currents are also $I_1$ as it is star connected. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_L)(I_L) = \sqrt{3}V_2(\sqrt{3}I_2) = 3V_2I_2 = 3s$ i.e. three times the kVA rating of each single phase transformer.

Phase voltage transformation ratio: $\frac{V_2}{V_1} = \frac{N_2}{N_1} = k$

Line voltage transformation ratio: $\frac{V_2}{\sqrt{3}V_1} = \frac{N_2}{\sqrt{3}N_1} = \frac{k}{\sqrt{3}}$

![Fig. 6.5 Star-Delta connections](image)

This transformer connection gives least secondary terminal voltage among the all types of connection. Commonly used in a step-down transformer. This transformer is generally used at the end of a transmission line. Wye connection on the HV side reduces insulation costs, the neutral point on the HV side can be grounded, stable with respect to unbalanced loads.

### 6.5.4 Delta- Star (□-Y) Connection

In this connection to apply a rated voltage to the primary terminals a line to line voltage of $V_1$ is supplied, so that the primary rated voltage $V_1$ is impressed across each of the primary coils of the individual transformer, as for delta connection both line and phase voltages are same. This ensures $V_2$ to be induced across each of the secondary coil and the line to line voltage in the secondary will also be $\sqrt{3}V_2$. Now we have to calculate how much load current or kVA can be
supplied by this bank of three phase transformers without over loading any of the single phase transformers. From the individual rating of each transformer, we know maximum allowable currents of primary and secondary windings are $I_1$ and $I_2$ respectively. Since secondary side is connected in star, line current and the winding currents (phase current) are same. When the current in secondary winding becomes $I_2$, the corresponding reflected phase current in the primary becomes $I_1$ and the line current becomes $\sqrt{3}I_1$. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_p)(I_p) = \sqrt{3}(\sqrt{3}V_2)I_2 = 3V_2I_2 = 3I$ i.e. three times the kVA rating of each single phase transformer.

$$\text{Phase voltage transformation ratio} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = k$$

$$\text{Line voltage transformation ratio} = \frac{\sqrt{3}V_2}{V_1} = \frac{\sqrt{3}N_2}{N_1} = \sqrt{3}k$$

![Fig. 6.6 Delta-Star connections](image)

For the same turns ratio and applied voltage this transformer connection provides highest secondary terminal voltage for which it is commonly used as a step-up transformer. This transformer is generally used at the beginning end of a transmission line.

Dy and Yd connection (without neutral connection) Behavior of the bank with mesh connection on one side is similar to the one discussed under Dd connection. The harmonic currents and drops and the departure of the flux density from sinusoidal are larger in the present case compared to Dd banks.

### 6.6 Basics of three phase connection

In a single phase transformer there are only two windings namely primary and secondary. However, in a 3-phase transformer there will be separately 3 primary and 3 secondary coils. So these 3 primary and secondary coils are to be properly connected so that the voltage level of a
balanced 3-phase supply may be changed to another 3-phase balanced system of different voltage level.

For transformer-A, primary terminals are marked as $A_1 A_2$ and the secondary terminals are marked as $a_1 a_2$. The markings are done in such a way that $A_1$ and $a_1$ represent the dot (•) terminals. Similarly terminals for B and C transformers are marked and shown in figure 6.7.

It may be noted that individually each transformer will work following the rules of single phase transformer i.e, induced voltage in $a_1 a_2$ will be in phase with applied voltage across $A_1 A_2$ and the ratio of magnitude of voltages and currents will be as usual decided by $a$ where $a = N_1 / N_2$, the turns ratio. This will be true for transformer-B and transformer-C as well i.e., induced voltage in $b_1 b_2$ will be in phase with applied voltage across $B_1 B_2$ and induced voltage in $c_1 c_2$ will be in phase with applied voltage across $C_1 C_2$.

**6.6.1 Proper Star Connection**

Now joining the terminals $A_2$, $B_2$ and $C_2$ of the 3 primary coils of the transformers and no connections are made between the secondary coils of the transformers. Now to the free terminals $A_1$, $B_1$ and $C_1$ a balanced 3-phase supply with phase sequence A-B-C is connected as shown in figure 6.8. Primary is said to be connected in star.
To impress a phase voltage of $V_1$ across each of primary coil the supply line voltage has to be $\sqrt{3}V_1$. However, the phasors $\overline{V}_{A_1A}$, $\overline{V}_{B_1B}$ and $\overline{V}_{C_1C}$ will have a mutual phase difference of $120^\circ$ as shown in figure 6.8. Then from the fundamental principle of single phase transformer we know, secondary coil voltage $\overline{V}_{a_1a}$ will be parallel to $\overline{V}_{A_1A}$, $\overline{V}_{b_1b}$ will be parallel to $\overline{V}_{B_1B}$ and $\overline{V}_{c_1c}$ will be parallel to $\overline{V}_{C_1C}$. Thus the secondary induced voltage phasors will have same magnitude $V_2$ but are displaced by $120^\circ$ mutually. Since the secondary coils are not connected, the secondary voltage phasors are shown independent without any connections between them.

Now if the secondary coil terminals $a_2$, $b_2$ and $c_2$ are joined together physically as shown in figure 6.9. So the secondary coil phasors should not be shown isolated as $a_2$, $b_2$ and $c_2$ become equipotential due to shorting of these terminals. Thus, the secondary coil voltage phasors should not only be parallel to the respective primary coil voltages but also $a_2$, $b_2$ and $c_2$ should be equipotential. Therefore, shift and place the phasors $\overline{V}_{a_2a}$, $\overline{V}_{b_2b}$ and $\overline{V}_{c_2c}$ in such a way that they remain parallel to the respective primary coil voltages and the points $a_2$, $b_2$ and $c_2$ are superposed.

![Fig. 6.9 Both primary and secondary are star connected](image)

An examination shows that either by connecting the secondary coil terminals $a_2$, $b_2$ and $c_2$ together or $a_1$, $b_1$ and $c_1$ together the secondary terminal voltages has a mutual phase displacement of $120^\circ$. Any other type attempt to connect in star will leads to an improper star connection.

### 6.6.2 Proper Delta Connection

To connect windings in delta, one should be careful enough to avoid dead short circuit. Suppose we want to carry out star / delta connection. The primary windings are connected by shorting $A_2$, $B_2$ and $C_2$ together as shown in the figure 6.10. As we know, in delta connection,
coils are basically connected in series and from the junction points, connection is made to supply load. Suppose we connect quite arbitrarily (without paying much attention to terminal markings and polarity), \(a_1\) with \(b_2\) and \(b_1\) with \(c_1\). As shown in the phasor diagram in figure 6.11, if a voltmeter is connected across S (i.e., between \(a_2\) and \(c_2\)), it is going to read the length of the phasor \(V_{a_2c_2}\) \((V_2 + 2V_2 \cos 60^0 = 2V_2)\). So if the switch’s’ is closed it is equivalent to put a short circuit across a voltage source which results in very large circulating current. So this type of connection should be avoided.

Another alternative way to attempt delta connection in the secondary could be: join \(a_1\) with \(b_2\) and \(b_1\) with \(c_2\). Before joining \(a_2\) with \(c_1\) to complete delta connection, examine the open circuit voltage \(V_{a_2c_1}\). Following the methods described before it can easily be shown that \(V_{a_2c_1} = 0\), which allows to join \(a_2\) with \(c_1\) without any circulating current. So this is a correct delta connection and is shown in figure 6.12. Although voltage exists in each winding, the resultant sum becomes zero as they are 120° mutually apart. The output terminals are taken from the junctions as \(a\), \(b\) and \(c\) for supplying 3-phase load. The corresponding phasor diagram is shown in figure 6.13.

Another valid delta connection on the LV side is also possible by joining \(a_2\) with \(b_1\), \(b_2\) with \(c_1\) and \(c_2\) with \(a_1\).

6.7 The per unit system for Three Phase Transformer
The process of solving circuits containing transformers using the referring method where all the different voltage levels on different sides of the transformers are referred to a common level, can be quite tedious.

The Per-unit System of measurements eliminates this problem. The required conversions are handled automatically by the method.

In per-unit system, each electrical quantity is measured as a decimal fraction of some base level. Any quantity can be expressed on a per-unit basis by the equation

\[ \text{Quantity per unit} = \frac{\text{actual value}}{\text{base value of quantity}} \]

Two base quantities are selected to define a given per-unit system. The ones usually selected are voltage and power. In a single phase system, the relationships are:

\[ P_{\text{base}}, Q_{\text{base}} \text{ or } S_{\text{base}} = V_{\text{base}} \times I_{\text{base}} \]

\[ R_{\text{base}}, X_{\text{base}}, Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \]

\[ Y_{\text{base}}, B_{\text{base}}, G_{\text{base}} = \frac{I_{\text{base}}}{V_{\text{base}}} \]

\[ Z_{\text{base}} = \left(\frac{V_{\text{base}}}{S_{\text{base}}}\right)^2 \]

and

\[ V_{\text{p.u.}} = \frac{V(\text{volt})}{V_{\text{base}}(\text{volt})} \]

\[ I_{\text{p.u.}} = \frac{I(\text{Amps})}{I_{\text{base}}(\text{Amps})} = \frac{I(\text{Amps})}{S_{\text{base}}}/V_{\text{base}} \]

\[ Z_{\text{p.u.}} = \frac{Z(\text{ohm})}{Z_{\text{base}}(\text{ohm})} = Z(\text{ohm}) \times \frac{I_{\text{base}}}{V_{\text{base}}} = Z(\text{ohm}) \times \frac{S_{\text{base}}}{(V_{\text{base}})^2} \]

In a power system, a base apparent power and voltage are selected at a specified point in the system. A transformer has no effect on the base apparent power of the system, since the apparent power equals the apparent power out.

Voltage changes as it goes through a transformer, so \( V_{\text{base}} \) changes at every transformer in the system according to its turns ratio. Thus, the process of referring quantities to a common level is automatically taken care of.
Many times, when more transformers are involved in a circuit one is required to choose a common base value for all of them. Parameters of all the machines are expressed on this common base. This is a common problem encountered in the case of parallel operation of two or more transformers. The conversion of the base values naturally lead to change in the per unit values of their parameters. An impedance \( Z_{p.u.\,old} \) on the old base of \( S_{base\,old} \) and \( V_{base\,old} \) shall get modified on new base \( S_{base\,new} \), \( V_{base\,new} \) as

\[
Z_{p.u.} = \left( \frac{Z_{p.u.\,old} \cdot V_{base\,old}^2}{S_{base\,old}} \right) \cdot \frac{S_{base\,new}}{V_{base\,new}^2}
\]

The term inside the bracket is nothing but the ohmic value of the impedance and this gets converted into the new per unit value by the new \( S_{base} \) and \( V_{base} \).

If all the equivalent circuit parameters are referred to the secondary side and per unit values of the new equivalent circuit parameters are computed with secondary voltage and current as the base values, there is no change in the per unit values. This can be easily seen by,

\[
Z_{p.u.} = Z_{ohm} \cdot \frac{S_{base}}{V_{base}^2} \text{ but } Z_{ohm} = \frac{1}{a^2} \cdot Z_{ohm}
\]

Where

- \( a \) - is the turns ratio of primary to secondary
- \( Z \) - impedance as seen by primary,
- \( Z' \) - impedance as seen by secondary.
- \( S_{base}' = S_{base} \) - as the transformer rating is unaltered.
- \( V_{base}' = \frac{V_{base}}{a} \)

From the above relationships it can be seen that \( Z_{p.u.} = Z_{p.u.} \).

Thus the per unit values help in dispensing away the scaling constants. The veracity of the parameters can be readily checked. Comparison of the parameters of the machines with those of similar ones throw in useful information about the machines. Comparing the efficiencies of two transformers at any load one can say that the transformer with a higher p.u. resistance has higher copper losses without actually computing the same.

Application of per unit values for the calculation of voltage regulation, efficiency and load sharing of parallel connected transformers will be discussed later at appropriate places.

**6.8 Open-Delta or V-V Connection**
When a three phase transformer bank connected in delta-delta supplies a balance 3-phase load, share the load equally. If one transformer is removed from the bank still it is possible to supply 3-phase power though at a reduced level, the resulting connection becomes open delta or V connection.

Fig. 6.14(i) shows open delta or V connection; one transformer is removed. As the three terminals are directly connected across a balanced supply voltage the vector sum of all the line voltages will be zero, i.e. \( \vec{V}_{AB} + \vec{V}_{BC} + \vec{V}_{CA} = 0 \)

Fig. 6.14(ii) shows the phasor diagram for voltages. Here \( V_{AB}, V_{BC} \) and \( V_{CA} \) represent the line-to-line voltages of the primary; \( V_{ab}, V_{bc} \) and \( V_{ca} \) represent line-to-line voltages of the secondary. The secondary line voltages \( V_{bc} \) and \( V_{ca} \) are directly available which are represented by solid line. The voltage across the open delta terminals ‘ab’ is indicated by dotted line \( V_{ab} \).

As the secondary terminal voltage \( V_{ab} \) forms the closing side of the secondary voltage triangle, it is similar to primary voltage triangle for which we can write \( V_{ab} + V_{bc} + V_{ca} = 0 \). Hence two transformers in open delta can deliver 3-phase power.

Fig. 6.14 (i) Open delta connection (ii) Phasor diagram for open delta

If \( |V_{bc}| = |V_{ca}| = V \) and with \( V_{ca} \) as the reference \( \vec{V}_{ca} = V \angle 0^0 \) and \( \vec{V}_{bc} = V \angle 120^0 \), \( \vec{V}_{ab} \) can be calculated as

\[
\vec{V}_{ab} = -(\vec{V}_{bc} + \vec{V}_{ca})
\]
\[ V_{ab} = -(V \angle 120^\circ + V \angle 0^\circ) \]
\[ V_{ab} = -[V(\cos 120^\circ + j \sin 120^\circ) + V] \]
\[ V_{ab} = -[V(\frac{1}{2} + j0.866) + V] \]
\[ V_{ab} = V(-\frac{1}{2} - j0.866) = V \angle -120^\circ \]

It shows under no load condition, the secondary line voltage form a balance 3-phase system of voltages.

**6.8.1 kVA delivered by open delta:**

Let \( V_{ph} \) and \( I_{ph} \) be the rated phase voltage and current respectively of each of the transformer.

**Case-I** in close delta

- Line voltage \( V_l = V_{ph} \)
- and Line current \( I_l = \sqrt{3} I_{ph} \)

VA delivered by the bank of transformers in delta = \( \sqrt{3} V_l I_l = \sqrt{3} V_{ph}(\sqrt{3} I_{ph}) = 3V_{ph}I_{ph} \)

**Case-II** in open delta

- Line voltage \( V_L = V_{ph} \)
- and Line current \( I_L = I_{ph} \)

VA delivered by the bank of transformers in delta = \( \sqrt{3} V_L I_L = \sqrt{3} V_{ph}I_{ph} \)

It is thus seen that the VA rating of open-delta is \( \sqrt{3} V_{ph}I_{ph} \) and not \( 2V_{ph}I_{ph} \).

\[
\frac{kVA \ in \ open \ delta}{kVA \ in \ close \ delta} = \frac{\sqrt{3} V_{ph}I_{ph}}{3V_{ph}I_{ph}} = 0.577 = 57.7\% 
\]

1. kVA supplied by each transformer = \( \frac{\sqrt{3} V_{ph}I_{ph}}{2} = 0.866 V_{ph}I_{ph} \), this implies each transformer is under loaded by a factor of 13.4%.

2. **utilisation factor or rating factor** = \( \frac{Actual \ available \ kVA}{Actual \ installed \ kVA \ in \ open \ delta} = \frac{\sqrt{3} V_{ph}I_{ph}}{2 V_{ph}I_{ph}} = 86.6\% \)

For example, three identical single-phase transformers, each of capacity 10 kVA, are connected in close delta. The total rating of the three transformers is 30 kVA. When one transformer is removed, the system reverts to V-V circuit and can deliver 3-phase power to a 3-phase load. However, the kVA capacity of the V-V circuit is reduced to \( 30 \times 0.577 = 17.3 \) kVA and not 20 kVA as might be expected. This reduced capacity can be determined in an alternate way. The
available capacity of the two transformers is 20 kVA. When operating in V-V circuit, only 86.6% of the rated capacity is available i.e. 20 x 0.866 = 17.3 kVA.

6.8.2 Power Factor of Transformers in V-V Circuit

When V-V circuit is delivering 3-phase power, the power factor of the two transformers is not the same (except at unity p.f.). Therefore, the voltage regulation of the two transformers will not be the same. If the load power factor angle is $\phi$, then,

- p.f. of transformer 1=$\cos(30^\circ - \phi)$
- p.f. of transformer 2=$\cos(30^\circ + \phi)$

i. When p.f is unity i.e. $\phi=0^\circ$, each transformer operates at the same p.f. of 0.866.

ii. When load p.f =0.866, i.e. $\phi=30^\circ$, one of the transformer operates at unity p.f. whereas the other at a p.f. of 0.5.

6.8.3 Applications of Open Delta or V-V Connection

The V-V circuit has a number of features that are advantageous. A few applications are given below by way of illustration:

i. The circuit can be employed in an emergency situation when one transformer in a complete $\triangle$-$\triangle$ circuit must be removed for repair and continuity of service is required.

ii. Upon failure of the primary or secondary of one transformer of a complete $\triangle$-$\triangle$ circuit, the system can be operated as V-V circuit and can deliver 3-phase power (with reduced capacity) to a 3-phase load.

iii. A circuit is sometimes installed as V-V circuit with the understanding that its capacity may be increased by adding one more transformer to form complete $\triangle$-$\triangle$ circuit. As discussed earlier.

6.9 Scott Connection or Three-phase to two-phase conversion

Scott connection is a type of circuit used to derive two-phase current from a three-phase source or vice-versa. The Scott three-phase transformer was invented by an engineer Charles F. Scott. In 1980s to bypass Thomas Edison’s rotary converter and thereby permit two-phase generator plants to drive Nikola Tesla’s three-phase motors.

At present though three phase power is mostly used, but for certain applications two-phase supplies are essential, such as

(i) Single-phase arc furnaces.
(ii) Low voltage single-phase rural application.
(iii) Electric traction
(iv) Two-phase control motors.

A two phase system of voltages consist of two equal voltages displaced from each other by a time phase angle of \(90^0\), so it cannot be taken from two separate single phase supplies.

The Scott connection uses two single-phase transformers of a special design to transform three-phase voltages and currents into two-phase voltages and currents.

1. The first transformer, called the ‘main transformer,’ has a center-tapped primary winding connected to the three-phase circuit with the secondary winding connected to the two-phase circuit. It is vital that the two halves of the center-tapped primary winding are wound around the same core leg so that the ampere-turns of the two halves cancel out each other. The ends of the center-tapped main primary winding are connected to two of the phases of the three-phase circuit.

2. The second transformer, called the ‘teaser transformer,’ has one end of its primary winding connected to the third phase of the three-phase circuit and the other end connected to the center tap of the primary winding of the main. If the main transformer has a turns ratio of 1:1, then the teaser transformer requires a turns ratio of 0.866:1 for balanced operation.

The Scott connection requires no primary neutral connection, so zero-sequence currents are blocked. The secondary windings of both the main and teaser transformers are connected to the two-phase circuit. The Scott connection is shown in figure 6.15 for a two-phase, five-wire circuit. The principle of operation of the Scott connection can be most easily seen by first applying a current to the teaser secondary windings, and then applying a current to the main secondary winding, calculating the primary currents separately and superimposing the results.
### 6.9.1 Principle of operation

The Scott Connected Transformer can convert a balanced three phase supply to two phase supply if the load on the two phase supply is balanced then the line current drawn from the three phase will be balanced. We will see how it is possible.

In the above diagram we have taken two single phase transformers. Let us say we have $N_1$, $N_2$ be the number of turns of the transformers. The first transformer is tapped by 50% so the number of turns of coil becomes $\frac{N_1}{2}$ on both sides of tapping. For the second transformer the supply voltage is applied to 86.6% of the primary no of turns i.e. $0.866N_1$. Then we supplied balanced three phase supply to $V_A$, $V_B$ and $V_C$ respectively. Let $V_1$ and $V_2$ be the induced voltages.

Now drawing the phasor $V_{BC}$, $V_{AB}$ & $V_{CA}$. Now the voltage phasor $V_{MC}$ will be half of $V_{BC}$ and voltage phasor $V_{MA}$ will be $V_{MC}+V_{CA}$. It shows that the voltage phasor $V_{AM}$ is right angle to the voltage phasor $V_{BC}$, hence, induced voltage $V_2$ will be in phase with the voltage $V_{AM}$ and its magnitude will be $\left(\frac{N_2}{0.866N_1}\right) * V_{MA}$. Voltage phasor $V_1$ will be in phase with the voltage $V_{BC}$ and its magnitude will be $\left(\frac{N_2}{N_1}\right) * V_{BC}$.
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Fig. 6.15 Phasor diagram of scott connected transformers (i) primary side (ii) Secondary side

From the phasor triangle

\[ V_{MA} = V_{MC} + V_{CA} \]

If \( V_{AB} = V_{BC} = V_{CA} = V_L \) (Linevoltage), then \( V_{BC} \) as the reference phasor \( V_{BC} = V_L[0^0] \),

\[ V_{CA} = V_L[120^0], \quad V_{AB} = V_L[-120^0], \quad V_{BM} = V_{MC} = \frac{1}{2} V_L[0^0]. \]

\( V_{MA} \) can be rewritten as

\[ V_{MA} = \frac{1}{2} V_L[0^0] + V_L[120^0] = \frac{1}{2} V_L + V_L (\cos 120^0 + j \sin 120^0) = j0.866V_L = 0.866V_L[90^0]. \]

Hence \( V_{MA} \), the voltage across the teaser is 0.866 of that across the main and leads it by 90\(^0\) as shown in the phasor diagram in figure 6.15. As \(|V_{MA}| = 0.866V_L\), so the magnitude of both the secondary voltages will be same i.e. be \( \frac{N_2}{N_1} \) *\( V_L \), and they are displaced in time phase by an angle of 90\(^0\) for which the secondary voltages are balanced two phase.

6.9.2 Calculation of Neutral

Fig. 6.16 Phasor diagram of scott connected transformers in primary side with neutral

The voltage between the lines and neutral point on the three phase side is \( \frac{V_L}{\sqrt{3}} = 0.578V_L \), which less than the teaser primary voltage is as it is 0.866\( V_L \). Hence the neutral must be on the teaser primary. The neutral point is, therefore, \( 0.866V_L - 0.578V_L = 0.288V_L \) from M. In other words from the phasor diagram

\[ |V_{MA}| = 0.866V_L, |V_{MN}| = 0.288V_L \] and \( |V_{NA}| = 0.578V \)
And the neutral divides the teaser primary \( MA \) in the ratio of \( V_{AN} : V_{MN} = 0.578 : 0.288 = 2:1. \)

Hence \( \frac{\text{Number of turns in } AN}{\text{Number of turns in } MN} = 2 \)

### 6.9.3 Load operation

For simplicity let us consider a balance two phase load connected at the secondaries. As the load is balance \( I_{ad} = I_{bc} = I_2 \quad \square \quad \square \) With \( I_{bc} \) as the reference, \( I_{bc} = I_2 \quad \square \quad \square \quad I_{ad} = jI_2 \)

The current in the secondary can be calculated by mmf balance.

Considering the teaser transformer,

\[
N_2 I_2 = \frac{\sqrt{3}}{2} N_1 I_A
\]

\[
\Rightarrow I_A = \frac{2}{\sqrt{3}} N_2 N_1 \frac{r}{I_2}
\]

Considering the main transformer,

\[
N_2 I_2 = \frac{N_1}{2} I_B - \frac{N_1}{2} I_C
\]

\[
\Rightarrow I_B - I_C = 2 \frac{N_2}{N_1} I_2 \quad \text{...............}(i)
\]

We know that

\[
I_A + I_B + I_C = 0
\]

\[
\Rightarrow I_B + I_C = -I_A \quad \text{...............}(ii)
\]

Solving equation i and ii, we have

\[
\Rightarrow I_B = \frac{N_2}{N_1} I_2 - \frac{1}{\sqrt{3}} \frac{N_2}{N_1} I_2 = I_2 - \frac{I_A}{2}
\]

and

\[
I_C = -\frac{N_2}{N_1} I_2 - \frac{1}{\sqrt{3}} \frac{N_2}{N_1} I_2 = -I_2 - \frac{I_A}{2}
\]
The magnitude of all the three phase currents are given by

\[ |I_A| = \frac{2}{\sqrt{3}} \frac{N_2}{N_1} |I_2| \]

\[ |I_B| = |I_C| = \sqrt{\left( \frac{N_2}{N_1} I_2 \right)^2 + \left( \frac{1}{\sqrt{3}} \frac{N_2}{N_1} I_2 \right)^2} = \frac{2}{\sqrt{3}} \frac{N_2}{N_1} |I_2| \]

Hence the magnitudes of all the three phases are same. But the input current to be balance ‘\( \alpha \)’ has to be 30°.

\[ \alpha = \tan^{-1} \left( \frac{1}{\sqrt{3}} \frac{N_2}{N_1} I_2 \right) = \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) = 30° \]

6.9.4 The Advantages of the Scott T configuration

1. If desired, a three phase, two phase, or single phase load may be supplied simultaneously
2. The neutral points can be available for grounding or loading purposes

6.9.5 The Disadvantages when used for 3 Phase Loading

1. This type of asymmetrical connection (3 phases, 2 coils), reconstructs three phases from 2 windings. This can cause unequal voltage drops in the windings, resulting in potentially unbalanced voltages to be applied to the load.
2. The transformation ratio of the coils and the voltage obtained may be slightly unbalanced due to manufacturing variances of the interconnected coils.
3. This design’s neutral has to be solidly grounded. If it is not grounded solidly, the secondary voltages could become unstable.
4. The inherent single phase construction and characteristics of this connection produces a comparatively bulky and heavier transformer when compared with a normal three phase transformer of the same rating.

6.10 Vector Group of 3-phase transformer connection

Three balanced 3-phase voltages can be connected in star or mesh fashion to yield a balanced 3-phase 3-wire system. The transformers that work on the 3-phase supply have star, mesh or zig zag connected windings on either primary secondary or both. In addition to giving different voltage ratios, they introduce phase shifts between input and output sides. These connections are broadly classified into 4 popular vector groups.

1. Group I: zero phase displacement between the primary and the secondary.
2. Group II: $180^\circ$ phase displacement.
3. Group III: $30^\circ$ lag phase displacement of the secondary with respect to the primary.
4. Group IV: $30^\circ$ lead phase displacement of the secondary with respect to the primary.

The secondary voltages of a 3-phase transformer may undergo a phase shift of $30^\circ$ leading or $30^\circ$ lagging or $0^\circ$ i.e, no phase shift or $180^\circ$ reversal with respective line or phase to neutral voltages. On the name plate of a three phase transformer, the vector group is mentioned. Typical representation of the vector group could be Yd$_1$ or Dy$_{11}$ etc. The first capital latter Y indicates that the primary is connected in star and the second lower case latter d indicates delta connection of the secondary side. The third numerical figure conveys the angle of phase shift based on clock convention. The minute hand is used to represent the primary phase to neutral voltage and always shown to occupy the position 12. The hour hand represents the secondary phase to neutral voltage and may, depending upon phase shift, occupy position other than 12 as shown in the figure 6.17.
6. 10 .1 Group I: zero phase displacement between the primary and the secondary.

The phase displacement between the primary and secondary respective line or phase to neutral voltages is zero for this group connections.

6. 10.1.1 Yy0 Connection

For Yy arrangement first the primary is connected in star with $A_1B_1C_1$ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.18(i), here $a_1a_2$ is parallel to $A_1A_2$. Similarly $b_1b_2$ and $c_1c_2$ are parallel to $B_1B_2$ and $C_1C_2$ respectively. In the secondary side if $a_1b_1c_1$ connected together then the line voltages $C_2B_2$ and $c_2b_2$ in primary and secondary side respectively coincide with each other and thus there is no phase angle between primary and secondary.

![Phasor diagram of Yy0 arrangement](image)

(ii)

Fig. 6.18 (i) Phasor diagram of Yy0 arrangement (ii) Connection diagram for it

6. 10.1.2 Dd0 Connection

For Dd arrangement first the primary is connected in delta with $A_2B_1$, $B_2C_1$ and $C_2A_1$ connected together and the secondary kept open. The corresponding phasor diagram are shown
in figure 6.19(i), here \(a_1a_2\) is parallel to \(A_1A_2\). Similarly \(b_1b_2\) and \(c_1c_2\) are parallel to \(B_1B_2\) and \(C_1C_2\) respectively. In the secondary side to have a phase displacement of \(0^\circ\), \(a_2b_1\), \(b_2c_1\) and \(c_2a_1\) connected together.

![Diagram](image1)

**Fig. 6.19 (i) Phasor diagram of Dd0 arrangement (ii) Connection diagram for it**

### 6.10.1.3 Dz0 Connection

A zigzag transformer is a special purpose transformer with a zigzag or 'interconnected star' winding connection, such that each output is the vector sum of two phases offset by \(120^\circ\). For such a transformer it requires two secondaries corresponds to each primary of exactly equal numbers of turns. The primary is connected in delta and the corresponding two set of secondary phasors are shown in the figure below. The phase voltage formed by the phasors \(b_1b_2\) and \(c_3c_4\) is parallel to \(C_1C_2\), making a phase displacement of \(0^\circ\) between the primary and secondary.

![Diagram](image2)
Fig. 6.20 (i) Phasor diagram of Dz0 arrangement (ii) Connection diagram for it

6.10.2 Group II: 180° phase displacement.

The phase displacement between the primary and secondary respective line or phase to neutral voltages is 180° for this group connections.

6.10.2.1 Yy6 Connection

For this arrangement first the primary is connected in star with $A_1B_1C_1$ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.21(i), here $a_1a_2$ is parallel to $A_1A_2$. Similarly $b_1b_2$ and $c_1c_2$ are parallel to $B_1B_2$ and $C_1C_2$ respectively. In the secondary side all the phasors are reversed with respect to the primary connections, so as to get a phase displacement of 180°. For which in the secondary side, terminals $a_2b_2c_2$ are connected together to form the neutral as shown in figure 6.21.
Fig. 6.21 (i) Phasor diagram of Yy0 arrangement (ii) Connection diagram for it

6. 10.2.2 Dd6 Connection

For this arrangement first the primary is connected in delta with $A_2B_1$, $B_2C_1$ and $C_2A_1$ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.22(i), here $a_1a_2$ is parallel to $A_1A_2$. Similarly $b_1b_2$ and $c_1c_2$ are parallel to $B_1B_2$ and $C_1C_2$ respectively. In the secondary side to have a phase displacement of $180^0$, $a_2c_1$, $b_2a_1$ and $c_2b_1$ are connected together.

Fig. 6.22 Phasor diagram of Dd6 arrangement (ii) Connection diagram for it

6. 10.2.3 Dz6 Connection

The primary is connected in delta and the corresponding two set of secondary phasors are shown in the figure below. The phase voltage formed by the phasors $b_1b_2$ and $c_2c_3$ is anti parallel to $C_2C_1$, making a phase displacement of $180^0$ between the primary and secondary.
6.10.3 Group III: 30° lag phase displacement of the secondary with respect to the primary.

A phase displacement of -30° means that the secondary line phase lags the corresponding primary phasor by 30° as shown in fig. below.

6.10.3.1 Yd1 Connection

For Yd arrangement first the primary is connected in star with A₁B₁C₁ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.24(i), here a₁a₂ is parallel to A₁A₂. Similarly b₁b₂ and c₁c₂ are parallel to B₁B₂ and C₁C₂ respectively. In the secondary has to be delta connected its line and phase voltages are same. The line voltage b₁b₂ is lagging by an angle 30° to the primary line voltage C₂B₂ phasor, c₁c₂, is lagging by an angle 30° to A₂C₂ and so on. For which the arrangement shown below provides a phase displacement of 30° lagging between the corresponding line voltages of primary and secondary.
6. 10.3.2 Dy1 Connection

For Dy arrangement first the primary is connected in delta with \( A_2B_1, B_2C_1 \) and \( C_2A_1 \) connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.25(i), here \( a_1a_2 \) is parallel to \( A_1A_2 \). Similarly \( b_1b_2 \) and \( c_1c_2 \) are parallel to \( B_1B_2 \) and \( C_1C_2 \) respectively. The line voltage \( c_2b_2 \), is lagging by an angle \( 30^0 \) to the primary line voltage \( C_1C_2 \) phasor, \( a_2c_2 \), is lagging by an angle \( 30^0 \) to \( A_1A_2 \) and so on. For which the arrangement shown below provides a phase displacement of \( 30^0 \) lagging between the corresponding line voltages of primary and secondary.
6.10.3.3 Yz1 Connection

For this arrangement first the primary is connected in star with A₂B₂C₂ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.26(i), here a₁a₂ and a₃a₄ is parallel to A₁A₂. Similarly b₁b₂, b₃b₄ and c₁c₂, c₃c₄ are parallel to B₁B₂ and C₁C₂ respectively. To have a phase displacement of 30° lagging between the corresponding phase voltages phasor c₃c₄ and a₁a₂ are connected in series, which forms the resultant phase voltage c₃a₁.
6.10.4 Group IV: 30° lead phase displacement of the secondary with respect to the primary.

A phase displacement of 30° means that the secondary line phase leads the corresponding primary phasor by 30° as shown in fig. below.

6. 10.4.1 Yd11 Connection

For Yd arrangement first the primary is connected in star with \(A_1B_1C_1\) connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.24(i), here \(a_1a_2\) is parallel to \(A_1A_2\). Similarly \(b_1b_2\) and \(c_1c_2\) are parallel to \(B_1B_2\) and \(C_1C_2\) respectively. In the secondary has to be delta connected its line and phase voltages are same. The line voltage \(c_1c_2\) is leading by an angle 30° to the primary line voltage \(C_2B_2\) phasor, \(a_1a_2\) leading by an angle 30° to \(A_2C_2\) and so on. For which the arrangement shown below provides a phase displacement of 30° leading between the corresponding line voltages of primary and secondary.

![Phasor diagram of Yd11 arrangement](image)

6.10.4.2 Dy11 Connection

![Connection diagram for Dy11](image)
For Dy arrangement first the primary is connected in delta with A₂C₁, B₂A₁ and C₂B₁ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.28(i), here a₁a₂ is parallel to A₁A₂. Similarly b₁b₂ and c₁c₂ are parallel to B₁B₂ and C₁C₂ respectively. The line voltage c₂b₂ is leading by an angle 30° to the primary line voltage B₁B₂ phasor, a₂b₂ leading by an angle 30° to A₁A₂ and so on. For which the arrangement shown below provides a phase displacement of 30° leading between the corresponding line voltages of primary and secondary.

6. 10.4.3 Yz11 Connection

For this arrangement first the primary is connected in star with A₂B₂C₂ connected together and the secondary kept open. The corresponding phasor diagram are shown in figure 6.26(i), here a₁a₂ and a₃a₄ is parallel to A₁A₂. Similarly b₁b₂, b₃b₄ and c₁c₂, c₃c₄ are parallel to B₁B₂ and C₁C₂ respectively. To have a phase displacement of 30° leading between the corresponding phase voltages phasor c₃c₄ and b₂b₁ are connected in series, which forms the resultant phase voltage c₃b₁.
6.11 Parallel operation of transformers

Parallel operation mean two or more transformers are connected to the same supply bus bars on the primary side and to a common bus bar/load on the secondary side. Such requirement is frequently encountered in practice. The reasons that necessitate parallel operation are as follows.

1. Non-availability of a single large transformer to fulfill the load demand.
2. To fulfill the future demand. The power demand might have increased over a time a number of transformers connected in parallel will then be pressed into service.
3. To improved reliability. Even if one of the transformers gets into a fault or is taken out for maintenance/repair the continuity of the service can be maintained.
4. To reduce the spare capacity. If many smaller size transformers are used one machine can be used as spare. If only one large machine is feeding the load, a spare of similar rating has to be available.

Fig. 6.29 Phasor diagram of Y11 arrangement (ii) Connection diagram for it
5. It may be easier to transport smaller ones to site and work them in parallel compared to a large unit.

6.11.1 Conditions to be fulfilled for parallel operation

There are some essential conditions which should be fulfilled before connecting transformers in parallel.

1. The polarity must be the same, so that there is no circulating current between the transformers.
2. The primary and secondary voltage rating of the transformer should be the same.
3. 3-phase transformers must belong to the same vector group.
4. The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.
5. The magnitudes of leakage impedances of the transformers should be inversely proportional to their KVA ratings for better load sharing. i.e. The per unit impedance of each machine on its own base must be the same.
6. The quality of the impedances i.e. $\frac{x}{r}$ ratio should be same.

Polarity must be same: Inside the loop formed by the two secondaries the resulting voltage must be zero. If it is wrongly connected, the secondary winding voltages will aid each other. Since the windings are already closed by virtue of the parallel connection, a situation like dead short circuit prevails resulting in large current.

Same voltage ratio: If the ratio is different, when the primaries are connected to same bus bars, the secondaries do not show the same voltage, paralleling them result in a circulating current between the secondaries. Reflected circulating current will be there on the primary side also. Thus even without connecting a load considerable current can be drawn by the transformers and they produce copper losses. In such cases the combined full load of the two transformers can never be met without one transformer getting overloaded.

Same vector group: A Y d1 transformer can be paralleled with a Dy1 transformer but not with a say, Y d11 transformer. Here also wrong selection of vector groups will essentially mean dead short circuit in the secondary due to difference in phases between the corresponding phases.
**Same phase sequence**: The phase sequence of operation becomes relevant only in the case of poly phase systems. If the phase sequences are not the same then the two transformers cannot be connected in parallel even if they belong to same vector group. If transformer connected with wrong phasor group due to difference in phases between the corresponding phases it leads to a dead short circuit. The phase sequence can be found out by the use of a phase sequence indicator.

**Same per unit impedance** Transformers of different ratings may be required to operate in parallel. If they have to share the total load in proportion to their ratings the larger machine has to draw more current. The voltage drop across each machine has to be the same by virtue of their connection at the input and the output ends. Thus the larger machines have smaller impedance and smaller machines must have larger ohmic impedance. Thus the impedances must be in the inverse ratios of the ratings. As the voltage drops must be the same the per unit impedance of each transformer on its own base, must be equal.

**Same quality of impedance**: The fulfillment of this condition means that the sum of the KVAs handled by the transformers will be equal to the load KVA. To understand this, let us look at the phasor diagrams shown in figure 6.30 and 6.31. Let $V_{XY}$ be the voltage phasor across the parallel branches of $Z_{ea}$ and $Z_{eb}$. Suppose the qualities of the impedances are different with power factor angles $\theta_A$ and $\theta_B$ respectively. The current phasors $I_a$ and $I_b$ can be drawn as shown in figure 6.31. Since the ratio of impedances are in the inverse proportion of the magnitudes of the impedances, then if $I_a$ is rated current, $I_b$ too will be rated. But in this case current supplied to the load, $I$ is less than the sum of the magnitudes of the currents $I_a$ and $I_b$ for obvious reason. In other words, although the individual transformers will be operating at their rated kVAs, kVA supplied to the load ($S = VI$) will be less than sum of kVAs of the transformers. Transformers in this case operate at different power factors.

However, if the qualities of the impedances are same, $I_a$ and $I_b$ will be in same phase and scalar sum of $I_a$ and $I_b$ will be equal to load current $I$. This ensures scalar sum of the kVAs, indeed is kVA supplied to the load and the transformers operate at the same power factors. A slight difference in the qualities of the impedances of the transformers can always allow.
Fig. 6.30: Phasor sum when the x/r ratios are different.

Fig. 6.31: Phasor sum when the x/r ratio is same.

Performance of two or more single phase transformers working in parallel can be computed using their equivalent circuit. In the case of poly phase banks also the approach is identical and the single phase equivalent circuit of the same can be used. Basically two cases arise in these problems.

Case A: when the voltage ratio of the two transformers is the same and Case B: when the voltage ratios are not the same.

These are discussed now in sequence.

6.11.2 Case A: Equal voltage ratios

Always two transformers of equal voltage ratios are selected for working in parallel. This way one can avoid a circulating current between the transformers. Load can be switched on subsequently to these bus bars. Neglecting the parallel branch of the equivalent circuit the above connection can be shown as in Fig. 6.32(a),(b). The equivalent circuit is drawn in terms of the secondary parameters. This may be further simplified as shown under Fig. 6.32(c).
Fig. 6.32

The voltage drop across the two transformers must be the same by virtue of common connection at input as well as output ends. By inspection the voltage equation for the drop can be written as

\[ I_A Z_A = I_B Z_B = IZ = \nu \quad \text{(say)} \]

Here \( I = I_A + I_B \)

and \( Z \) is the equivalent impedance of two transformers given by,

\[ Z = \frac{Z_A Z_B}{Z_A + Z_B} \]

Thus \( I_A = \frac{\nu}{Z_A} = \frac{IZ}{Z_A} = I_\cdot \frac{Z_B}{Z_A + Z_B} \) and \( I_B = \frac{\nu}{Z_B} = \frac{IZ}{Z_B} = I_\cdot \frac{Z_A}{Z_A + Z_B} \)

If the terminal voltage is \( V = IZ_L \) then the active power supplied by each of the two transformers is given by

\[ P_A = \text{Re}(VI_A^*) \text{ and } Q_A = \text{Im}(VI_A^*) \]
\[ P_B = \text{Re}(VI_B^*) \text{ and } Q_B = \text{Im}(VI_B^*) \]

From the above it is seen that the transformer with higher impedance supplies lesser load current and vice versa. If transformers of dissimilar ratings are paralleled the transformer with larger rating shall have smaller impedance as it has to produce the same drop as the other transformer, at a larger current. Thus the ohmic values of the impedances must be in the inverse ratio of the ratings of the transformers. \( I_A Z_A = I_B Z_B \), therefore \( I_\cdot \frac{I_A}{I_B} = \frac{Z_A}{Z_B} \).

Expressing the voltage drops in p.u basis, we aim at the same per unit drops at any load for the transformers. The per unit impedances must therefore be the same on their respective bases. Fig. 6.33 shows the phasor diagram of operation for these conditions. The drops are magnified and shown to improve clarity. It is seen that the total voltage drop inside the transformers is \( \nu \) but the currents \( I_A \) and \( I_B \) are forced to have a different phase angle due to the difference in the internal power factor angles \( \phi_A \) and \( \phi_B \).
This forces the active and reactive components of the currents drawn by each transformer to be different (even in the case when current in each transformer is the same). If we want them to share the load current in proportion to their ratings, their percentage (or p.u) impedances must be the same. In order to avoid any divergence and to share active and reactive powers also properly, $\phi_A = \phi_B$. Thus the condition for satisfactory parallel operation is that the p.u resistances and p.u reactance must be the same on their respective bases for the two transformers. To determine the sharing of currents and power either p.u parameters or ohmic values can be used.

6.11.3 Case B: Unequal voltage ratios

One may not be able to get two transformers of identical voltage ratio in spite of one’s best efforts. Due to manufacturing differences, even in transformers built as per the same design, the voltage ratios may not be the same. In such cases the circuit representation for parallel operation will be different as shown in Fig. 40. In this case the two input voltages cannot be merged to one, as they are different. The load brings about a common connection at the output side. $E_A$ and $E_B$
are the no-load secondary emf. \( Z_L \) is the load impedance at the secondary terminals. By inspection the voltage equation can be written as below:

\[
E_A = I_A Z_A + (I_A + I_B) Z_L = V + I_A Z_A \\
E_B = I_B Z_B + (I_A + I_B) Z_L = V + I_B Z_B
\]

Solving the two equations the expression for \( I_A \) and \( I_B \) can be obtained as

\[
I_A = \frac{E_A Z_B + (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)} \quad \text{and} \quad I_B = \frac{E_B Z_A + (E_B - E_A) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)}
\]

\( Z_A \) and \( Z_B \) are phasors and hence there can be angular difference also in addition to the difference in magnitude. When load is not connected there will be a circulating current between the transformers. The currents in that case can be obtained by putting \( Z_L = \inf \) (after dividing the numerator and the denominator by \( Z_L \)). Then,

\[
I_A = -I_B = \frac{(E_A - E_B)}{(Z_A + Z_B)}
\]

If the load impedance becomes zero as in the case of a short circuit, we have,

\[
I_A = \frac{E_A}{Z_A} \quad \text{and} \quad I_B = \frac{E_B}{Z_B}
\]

Instead of the value of \( Z_L \) if the value of \( V \) is known, the currents can be easily determined (from Eqns. 93) as

\[
I_A = \frac{E_A - V}{Z_A}, \quad I_B = \frac{E_B - V}{Z_B}
\]

If more than two transformers are connected across a load then the calculation of load currents following the method suggested above involves considerable amount of computational labor. A simpler and more elegant method for the case depicted in Fig. 41 is given below. It is known by the name parallel generator theorem.

\[
I_L = I_A + I_B
\]

But

\[
I_A = \frac{E_A - V}{Z_A}, \quad I_B = \frac{E_B - V}{Z_B} \\
V = I_L Z_L
\]
CHAPTER-7

Single Phase and Special Purpose Motors
Introduction to single phase induction motor

The characteristics of single phase induction motors are identical to 3-phase induction motors except that single phase induction motor has no inherent starting torque and some special arrangements have to be made for making it start itself. It follows that during starting period the single phase induction motor must be converted to a type which is not a single phase induction motor in the sense in which the term is ordinarily used and it becomes a true single phase induction motor when it is running and after the speed and torque have been raised to a point beyond which the additional device may be dispensed with. For these reasons, it is necessary to distinguish clearly between the starting period when the motor is not a single phase induction motor and the normal running condition when it is a single phase induction motor. The starting device adds to the cost of the motor and also requires more space. For the same output a 1-phase motor is about 30% larger than a corresponding 3-phase motor.

The single phase induction motor in its simplest form is structurally the same as a polyphase induction motor having a squirrel cage rotor, the only difference is that the single phase induction motor has single winding on the stator which produces mmf stationary in space but alternating in time, a polyphase stator winding carrying balanced currents produces mmf rotating in space around the air gap and constant in time with respect to an observer moving with the mmf. The stator winding of the single phase motor is disposed in slots around the inner periphery of a laminated ring similar to the 3-phase motor.

![Fig. 7.1: Elementary single phase induction motor](image)

An induction motor with a cage rotor and single phase stator winding is shown schematically in Fig. 7.1. The actual stator winding as mentioned earlier is distributed in slots so as to produce an approximately sinusoidal space distribution of mmf.

### PRINCIPLE OF OPERATION

Suppose the rotor is at rest and 1-phase supply is given to stator winding. The current flowing in the stator winding gives rise to an mmf whose axis is along the winding and it is a pulsating mmf, stationary in space and varying in magnitude, as a function of time, varying from positive
maximum to zero to negative maximum and this pulsating mmf induces currents in the short-circuited rotor of the motor which gives rise to an mmf. The currents in the rotor are induced due to transformer action and the direction of the currents is such that the mmf so developed opposes the stator mmf. The axis of the rotor mmf is same as that of the stator mmf. Since the torque developed is proportional to sine of the angle between the two mmf and since the angle is zero, the net torque acting on the rotor is zero and hence the rotor remains stationary.

For analytical purposes a pulsating field can be resolved into two revolving fields of constant magnitude and rotating in opposite directions as shown in Fig. 1.1 and each field has a magnitude equal to half the maximum length of the original pulsating phasor.

Fig.7.2. Representation of the pulsating field by space phasor.

These component waves rotate in opposite direction at synchronous speed. The forward (anticlockwise) and backward-rotating (clockwise) mmf waves’ f and b are shown in Fig. 7.2. In case of 3-phase induction motor there is only one forward rotating magnetic field and hence torque is developed and the motor is self-starting. However, in single phase induction motor each of these components mmf wave produces induction motor action but the corresponding torques is in opposite direction. With the rotor at rest the forward and backward field produce equal torques but opposite in direction and hence no net torque is developed on the motor and the motor remains stationary. If the forward and backward air gap fields remained equal when the rotor is revolving, each of the component fields would produce a torque-speed characteristic similar to that of a polyphase induction motor with negligible leakage impedance as shown by the dashed curves f and b in Fig. 7.3.

The resultant torque-speed characteristic which is the algebraic sum of the two component curves shows that if the motor were started by auxiliary means it would produce torque in whatever direction it was started.
In reality the two fields, forward and backward do not remain constant in the air gap and also the effect of stator leakage impedance can’t be ignored. In the above qualitative analysis the effects of induced rotor currents have not been properly accounted for.

When single phase supply is connected to the stator and the rotor is given a push along the forward rotating field, the relative speed between the rotor and the forward rotating magnetic field goes on decreasing and hence the magnitude of induced currents also decreases and hence the mmf due to the induced current in the rotor decreases and its opposing effect to the forward rotating field decreases which means the forward rotating field becomes stronger as the rotor speeds up. However for the backward rotating field the relative speed between the rotor and the backward field increases as the rotor rotates and hence the rotor emf increases and hence the mmf due to this component of current increases and its opposing effect to the backward rotating field increases and the net backward rotating field weakens as the rotor rotates along the forward rotating field. However, the sum of the two fields remains constant since it must induce the stator counter emf which is approximately constant if the stator leakage impedance drop is negligible. Hence, with the rotor in motion the torque of the forward field is greater and that of the backward field is shown in Fig. 7.4.

Fig. 7.4. Torque-speed characteristic of a 1-phase induction motor taking into account changes in the flux waves.
In the normal running region at a few per cent slip the forward field is several times stronger than the backward field and the flux wave does not differ materially from the constant amplitude revolving field in the air gap of a balanced polyphase motor. Therefore, in the normal running range of the motor, the torque-speed characteristic of a single phase motor is not very much different from that of a polyphase motor having the same rotor and operating with the same maximum air gap flux density.

Double-stator frequency torque pulsation are produced by the interaction of the oppositely rotating flux and mmf waves which move past each other at twice synchronous speed. These double frequency torques produce no average torque as these pulsations are sinusoidal and over the complete cycle the average torque is zero. However, sometimes these are additive to the main torque and for another half a cycle these are subtractive and therefore a variable torque acts on the shaft of the motor which makes the motor noisier as compared to a polyphase induction motor where the total torque is constant. Such torque pulsations are unavoidable in single phase circuits. Mathematically

\[ T \propto I^2 \]
\[ I = I_m \sin \omega t \]
\[ T = K I^2 \sin^2 \omega t \]
\[ = K I_m^2 (1 - \cos \omega t)/2 \]

So the expression for torque contains a constant term superimposed over by a pulsating torque with pulsation frequency twice the supply frequency.

**STARTING OF SINGLE PHASE INDUCTION MOTORS**

The single phase induction motors are classified based on the method of starting method and in fact are known by the same name descriptive of the method. Appropriate selection of these motors depends upon the starting and running torque requirements of the load, the duty cycle and limitations on starting and running current drawn from the supply by these motors. The cost of single phase induction motor increases with the size of the motor and with the performance such as starting torque to current ratio (higher ratio is desirable), hence, the user will like to go in for a smaller size (hp) motor with minimum cost, of course, meeting all the operational requirements. However, if a very large no. of fractional horsepower motors are required, a specific design can always be worked out which might give minimum cost for a given performance requirements. Following are the starting methods.

(a) **Split-phase induction motor.** The stator of a split phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90°electrical as shown in Fig. The auxiliary winding is made of thin wire (super enamel copper wire) so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. Since the two windings are connected across the supply the current \( I_m \) and \( I_a \) in the main winding and auxiliary winding lag behind the supply voltage \( V \), \( I_a \) is leading the current \( I_m \) shown in Fig. This means the current through auxiliary winding reaches maximum value first and the mmf or flux due to \( I_a \) lies along the axis of the auxiliary winding and after
some time \((t = \theta/w)\) the current \(I_m\) reaches maximum value and the mmf or flux due to \(I_m\) lies along the main winding axis. Thus the motor becomes a 2-phase unbalanced motor. It is unbalanced since the two currents are not exactly 90° apart. Because of these two fields a starting torque is developed and the motor becomes a self-starting motor. After the motor starts, the auxiliary winding is disconnected usually by means of centrifugal switch that operates at about 75% of synchronous speed. Finally the motor runs because of the main winding. Since this being single phase some level of humming noise is always associated with the motor during running. A typical torque speed characteristic is shown in Fig. 7.5. It is to be noted that the direction of rotation of the motor can be reversed by reversing the connection to either the main winding or the auxiliary windings.

![Diagram of Split Phase Induction Motor](image)

Fig. 7.5: Split phase induction motor  (a) Connection (b) Phasor diagram at starting, (c) typical torque-speed characteristic.

**(b) Capacitor starts induction motor.**

Capacitors are used to improve the starting and running performance of the single phase inductions motors.

The capacitor start induction motor is also a split phase motor. The capacitor of suitable value is connected in series with the auxiliary coil through a switch such that \(I_a\) the current in the auxiliary coil leads the current \(I_m\) in the main coil by 90° in time phase so that the starting torque is maximum for certain values of \(I_a\) and \(I_m\). This becomes a balanced 2-phase motor if the magnitude of \(I_a\) and \(I_m\) are equal and are displaced in time phase by 90°. Since the two windings are displaced in space by 90° as shown in Fig. 7.6 maximum torque is developed at start. However, the auxiliary winding and capacitor are disconnected after the motor has picked up 75% of the synchronous speed. The motor will start without any humming noise. However, after the auxiliary winding is disconnected, there will be some humming noise.
Fig. 7.6. Capacitor start motor (a) Connection (b) Phasor diagram at start (c) Speed torque curve.

Since the auxiliary winding and capacitor are to be used intermittently, these can be designed for minimum cost. However, it is found that the best compromise among the factors of starting torque, starting current and costs results with a phase angle somewhat less than 90° between Im and Ia. A typical torque-speed characteristic is shown in Fig. high starting torque being an outstanding feature.

(c) Permanent-split capacitor motor. In this motor the auxiliary winding and capacitor are not disconnected from the motor after starting, thus the construction is simplified by the omission of the switch as shown in Fig. 7.7

Fig. 7.7: Permanent-split capacitor motor

Here the auxiliary winding and capacitor could be so designed that the motor works as a perfect 2-phase motor at anyone desired load. With this the backward rotating magnetic field would be
completely eliminated. The double stator frequency torque pulsations would also be eliminated; thereby the motor starts and runs as a noise free motor. With this there is improvement in p.f. and efficiency of the motor. However, the starting torque must be sacrificed as the capacitance is necessarily a compromise between the best starting and running characteristics. The torque-speed characteristic of the motor is shown in Fig.

(d) Capacitor start, capacitor run motor: If two capacitors are used with the auxiliary winding (as shown in Fig. 7.8) one for starting and other during the start and run, theoretically optimum starting and running performance can both be achieved.

![Fig. 7.8](image)

Fig. 7.8: (a) Capacitor start capacitor run motor (b) Torque-speed characteristic.

The small value capacitor required for optimum running conditions is permanently connected in series with the auxiliary winding and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor. The starting capacitor is disconnected after the motor starts.

(e) Shaded pole induction motor: (Fig. 7.9 shows schematic diagram of shaded pole induction motor.) The stator has salient poles with one portion of each pole surrounded by a short-circuited turn of copper called a shading coil. Induced currents in the shading coil (acts as an inductor) cause the flux in the shaded portion of the pole to lag the flux in the other portion. Hence the flux under the unshaded pole leads the flux under the shaded pole which results in a rotating field moving in the direction from unshaded to the shaded portion of the pole and a low starting torque is produced which rotates the rotor in the direction from unshaded to the shaded pole (A typical torque speed characteristic). The efficiency is low. These motors are the least expensive type of fractional horse power motor and are built up to 1/20 hp. since the rotation of the motor is in the direction from unshaded towards the shaded part of the pole, a shaded pole motor can be reversed only by providing two sets of shading coils which may be opened and closed or it may be reversed permanently by inverting the core.
LECTURE NOTES ON ELECTRICAL MACHINES-II

**Fig. 7.9.** Shaded pole motor and typical torque-speed characteristic

**Reluctance motor**

The *variable reluctance motor* is based on the principle that an unrestrained piece of iron will move to complete a magnetic flux path with minimum *reluctance*, the magnetic analog of electrical resistance. (Fig. below)

Synchronous reluctance

If the rotating field of a large synchronous motor with salient poles is de-energized, it will still develop 10 or 15% of synchronous torque. This is due to variable reluctance throughout a rotor revolution. There is no practical application for a large synchronous reluctance motor. However, it is practical in small sizes.

If slots are cut into the conductor less rotor of an induction motor, corresponding to the stator slots, a *synchronous reluctance motor* results. It starts like an induction motor but runs with a small amount of synchronous torque. The synchronous torque is due to changes in reluctance of the magnetic path from the stator through the rotor as the slots align. This motor is an inexpensive means of developing a moderate synchronous torque. Low power factor, low pull-out torque, and low efficiency are characteristics of the direct power line driven variable reluctance motor. Such was the status of the variable reluctance motor for a century before the development of semiconductor power control.

Switched reluctance

If an iron rotor with poles, but without any conductors, is fitted to a multi-phase stator, a *switched reluctance motor*, capable of synchronizing with the stator field results. When a stator coil pole pair is energized, the rotor will move to the lowest magnetic reluctance path. (Fig. 7.10 below) A switched reluctance motor is also known as a variable reluctance motor. The reluctance of the rotor to stator flux path varies with the position of the rotor.
Fig. 7.10: Reluctance is a function of rotor position in a variable reluctance motor.

Sequential switching (Fig. 7.11) of the stator phases moves the rotor from one position to the next. The magnetic flux seeks the path of least reluctance, the magnetic analog of electric resistance. This is an over simplified rotor and waveforms to illustrate operation.

Fig. 7.11: Sequential switching of stator phases of the reluctance motor

If one end of each 3-phase winding of the switched reluctance motor is brought out via a common lead wire, we can explain operation as if it were a stepper motor (Fig. 7.11). The other coil connections are successively pulled to ground, one at a time, in a wave drive pattern. This attracts the rotor to the clockwise rotating magnetic field in \(60^\circ\) increments.

Various waveforms may drive variable reluctance motors. (Fig. 7.12) Wave drive (a) is simple, requiring only a single ended unipolar switch. That is, one which only switches in one direction. More torque is provided by the bipolar drive (b), but requires a bipolar switch. The power driver must pull alternately high and low. Waveforms (a & b) are applicable to the stepper motor version of the variable reluctance motor. For smooth vibration free operation the 6-step approximation of a sine wave (c) is desirable and easy to generate. Sine wave drive (d) may be generated by a pulse width modulator (PWM), or drawn from the power line.
Doubling the number of stator poles decreases the rotating speed and increases torque. This might eliminate a gear reduction drive. A variable reluctance motor intended to move in discrete steps, stop, and start is a variable reluctance stepper motor, covered in another section. If smooth rotation is the goal, there is an electronic driven version of the switched reluctance motor. Variable reluctance motors or steppers actually use rotors like those in Fig. 7.13.

**Electronic driven variable reluctance motor**

Variable reluctance motors are poor performers when direct power line driven. However, microprocessors and solid state power drive makes this motor an economical high performance solution in some high volume applications.

Though difficult to control, this motor is easy to spin. Sequential switching of the field coils creates a rotating magnetic field which drags the irregularly shaped rotor around with it as it seeks out the lowest magnetic reluctance path. The relationship between torque and stator current is highly nonlinear—difficult to control.
**Electronic driven variable reluctance motor.**

An electronic driven variable reluctance motor (Fig. 7.14) resembles a brushless DC motor without a permanent magnet rotor. This makes the motor simple and inexpensive. However, this is offset by the cost of the electronic control, which is not nearly as simple as that for a brushless DC motor. Electronic control makes it practical to drive the motor well above and below the power line frequency. A variable reluctance motor driven by a *servo*, an electronic feedback system, controls torque and speed, minimizing ripple torque (Fig. 7.14).

![Variable Reluctance Motor Diagram](diagram)

**Fig. 7.14:** Electronic driven variable reluctance motor.

This is the opposite of the high ripple torque desired in stepper motors. Rather than a stepper, a variable reluctance motor is optimized for continuous high speed rotation with minimum ripple torque. It is necessary to measure the rotor position with a rotary position sensor like an optical or magnetic encoder, or derive this from monitoring the stator back EMF. A microprocessor performs complex calculations for switching the windings at the proper time with solid state devices. This must be done precisely to minimize audible noise and ripple torque. For lowest ripple torque, winding current must be monitored and controlled. The strict drive requirements make this motor only practical for high volume applications like energy efficient vacuum cleaner motors, fan motors, or pump motors. One such vacuum cleaner uses a compact high efficiency electronic driven 100,000 rpm fan motor. The simplicity of the motor compensates for the drive electronics cost. No brushes, no commutator, no rotor windings, no permanent magnets, simplifies motor manufacture. The efficiency of this electronic driven motor can be high. But, it requires considerable optimization, using specialized design techniques, which is only justified for large manufacturing volumes.

**Advantages**

- Simple construction- no brushes, commutator, or permanent magnets, no Cu or Al in the rotor.
- High efficiency and reliability compared to conventional AC or DC motors.
- High starting torque.
- Cost effective compared to brushless DC motor in high volumes.
- Adaptable to very high ambient temperature.
- Low cost accurate speed control possible if volume is high enough.

**Disadvantages**

- Current versus torque is highly nonlinear
- Phase switching must be precise to minimize ripple torque
- Phase current must be controlled to minimize ripple torque
- Acoustic and electrical noise
- Not applicable to low volumes due to complex control issues
Stepper Motors

STEPPER MOTOR – an electromagnetic actuator. It is an incremental drive (digital) actuator and is driven in fixed angular steps.

This means that a digital signal is used to drive the motor and every time it receives a digital pulse it rotates a specific number of degrees in rotation.

- Each step of rotation is the response of the motor to an input pulse (or digital command).
- Step-wise rotation of the rotor can be synchronized with pulses in a command-pulse train, assuming that no steps are missed, thereby making the motor respond faithfully to the pulse signal in an open-loop manner.
- Stepper motors have emerged as cost-effective alternatives for DC servomotors in high-speed, motion-control applications (except the high torque-speed range) with the improvements in permanent magnets and the incorporation of solid-state circuitry and logic devices in their drive systems.
- Today stepper motors can be found in computer peripherals, machine tools, medical equipment, automotive devices, and small business machines, to name a few applications.

Stepper motors are usually operated in open loop mode.

DC MOTORS VS. STEPPER MOTORS

- Stepper motors are operated open loop, while most DC motors are operated closed loop.
- Stepper motors are easily controlled with microprocessors; however logic and drive electronics are more complex.
- Stepper motors are brushless and brushes contribute several problems, e.g., wear, sparks, electrical transients.
- DC motors have a continuous displacement and can be accurately positioned, whereas stepper motor motion is incremental and its resolution is limited to the step size.
- Stepper motors can slip if overloaded and the error can go undetected. (A few stepper motors use closed-loop control.)
- Feedback control with DC motors gives a much faster response time compared to stepper motors.

ADVANTAGES OF STEPPER MOTORS

- Position error is noncumulative. A high accuracy of motion is possible, even under open-loop control.
- Large savings in sensor (measurement system) and controller costs are possible when the open-loop mode is used.
- Because of the incremental nature of command and motion, stepper motors are easily adaptable to digital control applications.
No serious stability problems exist, even under open-loop control. Torque capacity and power requirements can be optimized and the response can be controlled by electronic switching. Brushless construction has obvious advantages.

**DISADVANTAGES OF STEPPER MOTORS**

- They have low torque capacity (typically less than 2,000 oz-in) compared to DC motors.
- They have limited speed (limited by torque capacity and by pulse-missing problems due to faulty switching systems and drive circuits).
- They have high vibration levels due to stepwise motion.
- Large errors and oscillations can result when a pulse is missed under open-loop control.

**STEPPER MOTOR BASICS**

The above figure (7.15) is the cross-section view of a single-stack variable-reluctance motor. The stator core is the outer structure and has six poles or teeth. The inner device is called the rotor and has four poles. Both the stator and rotor are made of soft steel. The stator has three sets of windings as shown in the figure. Each set has two coils connected in series. A set of windings is called a “phase”. The motor above, using this designation, is a three-phase motor. Current is supplied from the DC power source to the windings via the switches I, II, and, III.

Starting with state (1) in the upper left diagram, note that in state (1), the winding of Phase I is supplied with current through switch I. This is called in technical terms, “phase I is excited”. Arrows on the coil windings indicate the magnetic flux, which occurs in the air-gap due to the excitation. In state I, the two stator poles on phase I being excited are in alignment with two of the four rotor teeth. This is an equilibrium state.
Next, switch II is closed to excite phase II in addition to phase I. Magnetic flux is built up at the stator poles of phase II in the manner shown in state (2), the upper right diagram. A counterclockwise torque is created due to the “tension” in the inclined magnetic flux lines. The rotor will begin to move and achieve state (3), the lower left diagram. In state (3) the rotor has moved 15°. When switch I is opened to de-energize phase I, the rotor will travel another 15° and reach state (4). The angular position of the rotor can thus be controlled in units of the step angle by a switching process. If the switching is carried out in sequence, the rotor will rotate with a stepped motion; the switching process can also control the average speed.

### Step Angle

The step angle, the number of degrees a rotor will turn per step, is calculated as follows:

\[
\text{Step Angle} (\Theta_s) = \frac{360^\circ}{S} = \frac{360^\circ}{mN_r}
\]

- \(m\) = number of phases
- \(N_r\) = number of rotor teeth

For this motor:

- \(m = 3\)
- \(N_r = 4\)
- \(S = mN_r = 3 \times 4 = 12\)
- \(\Theta_s = \frac{360^\circ}{12} = 30^\circ\ per\ step\)

### Basic wiring diagram:

![Basic wiring diagram](image.png)

Fig. 7.16: two phase stepper motor wiring diagram.
The step angle, the number of degrees a rotor will turn per step, is calculated as follows:

**TWO PHASE STEPPER-MOTOR WIRING DIAGRAM**

The above motor is a two-phase motor. This is sometimes called UNIPOLAR. The two-phase coils are center-tapped and in this case they the center-taps are connected to ground. The coils are wound so that current is reversed when the drive signal is applied to either coil at a time. The north and south poles of the stator phases reverse depending upon whether the drive signal is applied to coil 1 as opposed to coil 2.

**STEP SEQUENCING**

There are three modes of operation when using a stepper motor. The mode of operation is determined by the step sequence applied. The three step sequences are:

Wave

Full H = HIGH = +V

Half Stepping L = LOW = 0V

**WAVE STEPPING**

The wave stepping sequence is shown below.

<table>
<thead>
<tr>
<th>STEP</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
</tbody>
</table>

Wave stepping has less torque then full stepping. It is the least stable at higher speeds and has low power consumption.

**FULL STEPPING**

The full stepping sequence is shown below.

<table>
<thead>
<tr>
<th>STEP</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
</tbody>
</table>
HALF-STEPPING – A COMBINATION OF WAVE AND FULL STEPPING

The half-step sequence is shown below.

<table>
<thead>
<tr>
<th>STEP</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>5</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>6</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>7</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>8</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
</tbody>
</table>

The half-step sequence has the most torque and is the most stable at higher speeds. It also has the highest resolution of the main stepping methods. It is a combination of full and wave stepping.

Universal motor

Generally, the electric motors are operated either in DC Power or AC Power. But for some specific applications, it is desirable to introduce a motor that operates on either ac or dc supply. The word ‘Universal’ signifies that something which is compatible with versatile inputs. We have built small series motors up to ½ KW rating which operates on single phase ac supply as well as on dc. Such motors are called universal motors. An universal motor is a specifically designed series wound motor, that operates at approximately the same speed and output on either ac or dc voltage. In case of universal motor, the speed of rotation is slightly lesser when operating in AC. Because, the reactance voltage drop is present on ac but not on dc. So, the motor speed is somewhat lower for same load in ac operation than dc. This takes place especially at high loads. Most universal motors are designed to operate at speeds exceeding 3500 rpm. We will explain discuss the construction of this type of motor.

Construction of Universal Motor

![Universal motor diagram](image-url)
Basically, an **universal motor** is similar to a dc series motor in construction. However it is constructed with few series field turns, laminated armature and field circuits, low reluctance magnetic path, increased armature conductors and commutator segments. The frames of universal motor are usually made up of aluminum, rolled steel and cast iron. The commutation on ac is much poorer than on dc, due to current induced in the short circuited armature coils. If we use wide brushes then the short circuited current is excessive and motor starting torque is reduced. Brushes used are high resistance carbon ones so as to aid commutation. Compensating windings which are used in most large rating motors to improve commutation are not used in universal motors. In this motor armature current is quite small which cannot cause any commutation problems.

**Types of universal motors**

Universal motors are of two types. They are:

- **Compensated type (distributed field)**: It is of again two types namely:
  
  (i) Single field compensated motor – it resembles the stator winding of 2 pole, split phase ac motor.

  (ii) Two-field compensated motor – it has stator winding which consists of main winding and a compensated winding spaced 90 electrical degrees apart.

- **Uncompensated type (concentrated field)**: It is used for higher speeds and smaller output ratings. Concentrated field type is usually a salient pole machine. Laminated core having either straight or skewed slots or commutator are also present. The uncompensated motor is simpler and less expensive than compensated motor but gives poor speed regulation.

**Working principle of universal motor**

Now let us discuss the **operation** of this kind of motor in brief. In series circuit, same amount of current flows through all components. Similarly in a series wound motor, the same current flows through field windings and armature winding both. In an **universal motor**, both windings connected in series with each other. When the motor is supplied from ac or dc supply, magnetic fields are developed around the armature winding and field winding. They react on each other to produce an unidirectional torque forcefully. In some other words, the interaction in between series magnetic field and armature field causes to develop a torque and this torque leads to rotate the shaft. However a series motor which is specifically designed for dc operation suffers from following drawbacks when it is used on single phase ac supply:-

- Its efficiency is low due to hysteresis and eddy current losses.
- The p.f is low due to large reactance of the field and armature windings.
- The sparking at the brushes is excessive.
Characteristics of Universal Motor

The torque – speed \((\tau - N)\) characteristics of an universal motor is quite similar to that of a series wound dc motor. It has high starting torque at low speed and low starting torque at high speed. In small series motors losses are large at no-load to restrict the speed to a definite value. A centrifugal switch is placed on the motor shaft. The tension of the springs of the switch is adjusted so that the switch opens at a pre determined speed. An external resistor \(R\) is placed in series with armature to reduce the speed. When speed drops due to increase of load, the switch contacts close thereby shorting the resistor \(R\) thus raising the speed. Universal motors are high speed, small size motors as compared to other motors of same output. Here the full load power factor is high (0.9).

**Speed control of universal motor**

Speed control of universal motor is best obtained by solid-state devices. Since the speed of these motors is not limited by the supply frequency and may be as high as 20,000 rpm, they are most suitable for applications requiring high speeds. The factors that determine the speed for any dc motor are the same as those for ac series or universal motors i.e. flux and generated voltage. Generated voltage change is rarely employed in speed control method. Instead line voltage is varied .This has been accomplished by means of tapped resistor, rheostat in series with the line. Another method is by using a tapped field, thereby reducing the flux and hence raising the speed. This can be achieved by any one of the methods that follow:

- By using field poles wound in various sections with wires of different size and bringing out the taps from each section.
- By using tapped nichrome wires coils wound over a single field pole. In this method torque decreases with increase in speed.

**Applications of Universal Motor**

Universal motors find its applications in various devices. These are:

- The very small power output rating universal motors, which usually does not exceed 5 to10 watts are employed in equipments such as sewing machines, fans, portable hand tools, hair dryers, motion picture projectors and electric shavers.
- The higher rating (5-500 W) universal motor are used in vacuum cleaners, food mixers, blenders, cameras and calculating machines.
- This type of machine is used in table fans, polishers, portable drills and other kitchen appliances.